Two-Round Threshold Lattice-Based Signatures from Threshold Homomorphic Encryption

Kamil Doruk Gur (UMD),
Jonathan Katz (Google, UMD),
Tjerand Silde (NTNU)

Overview: Digital Signatures

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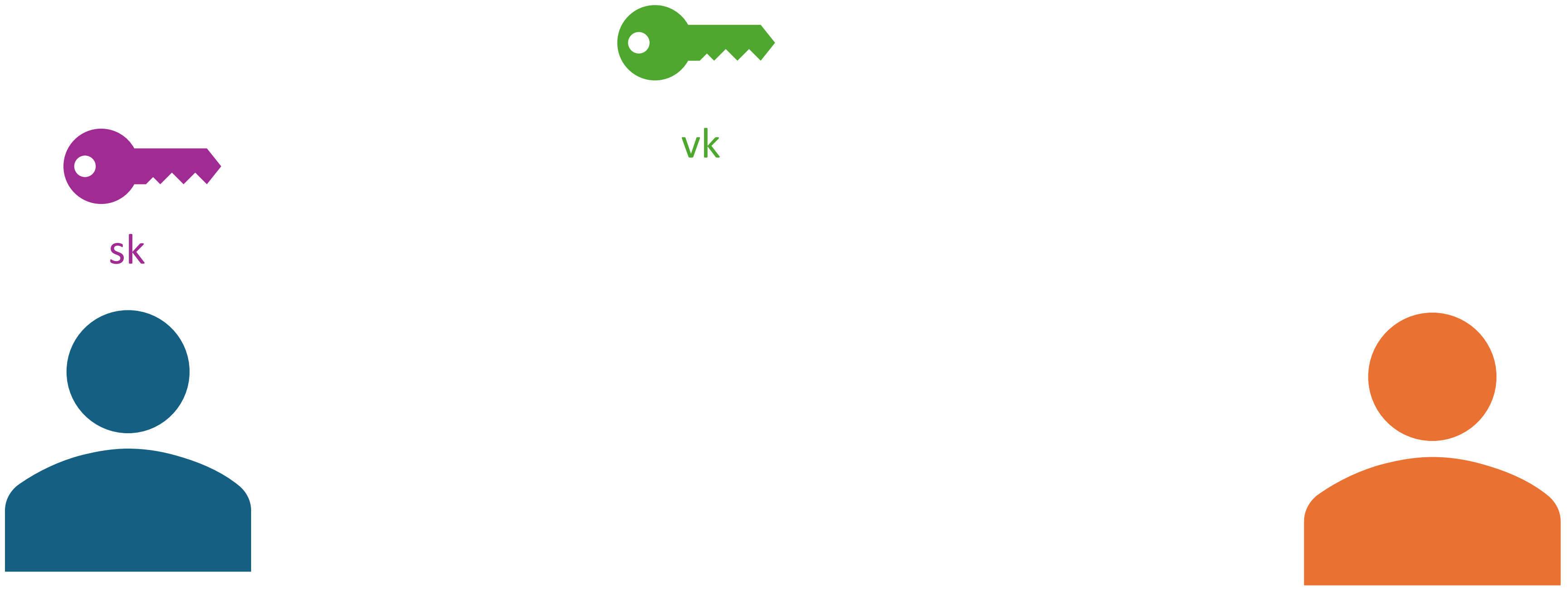

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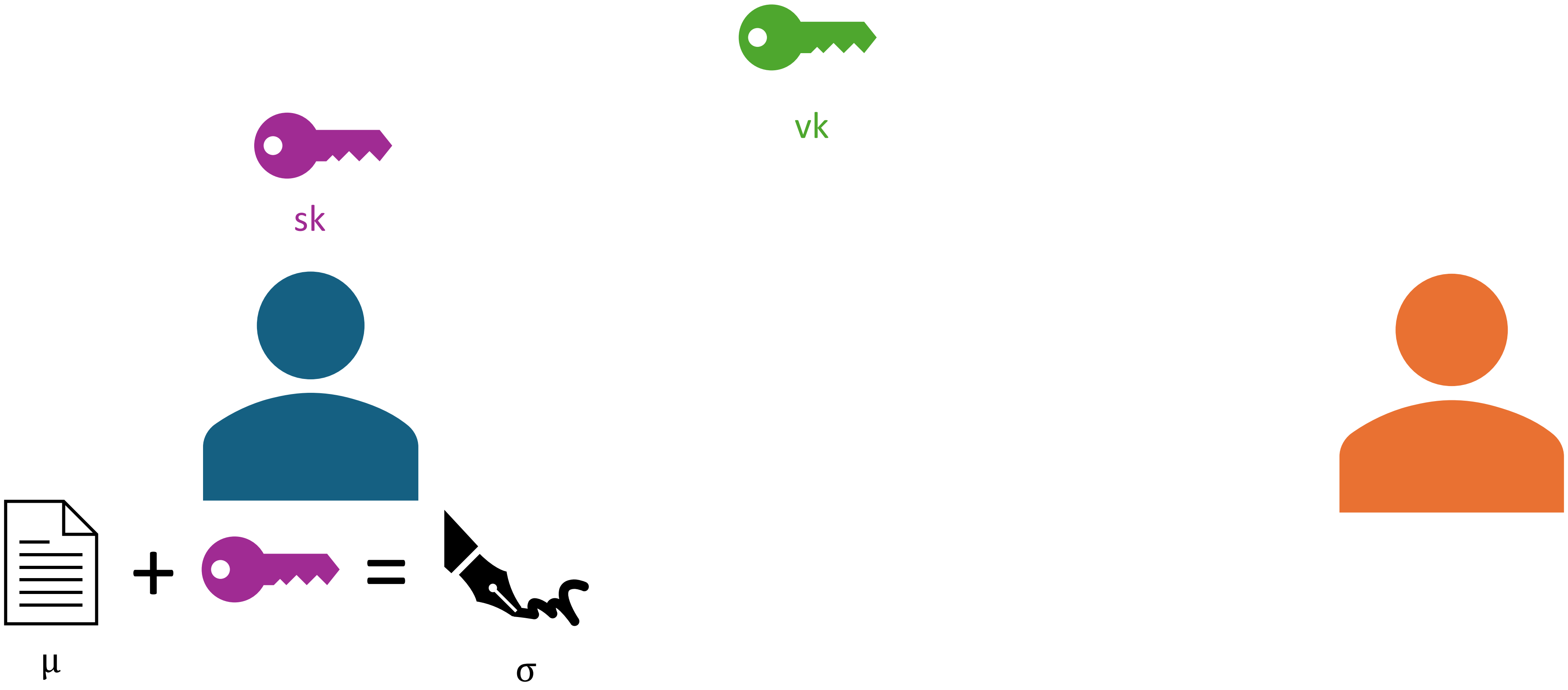

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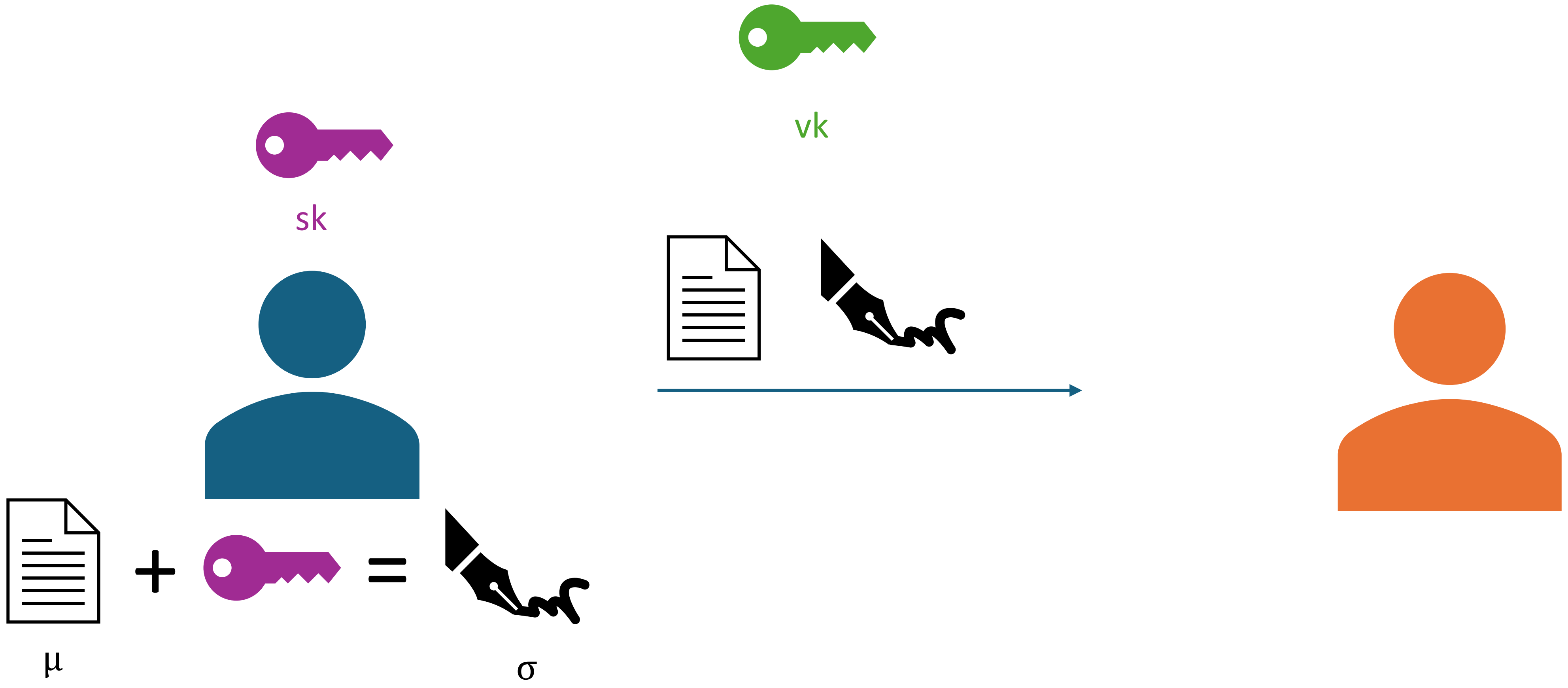

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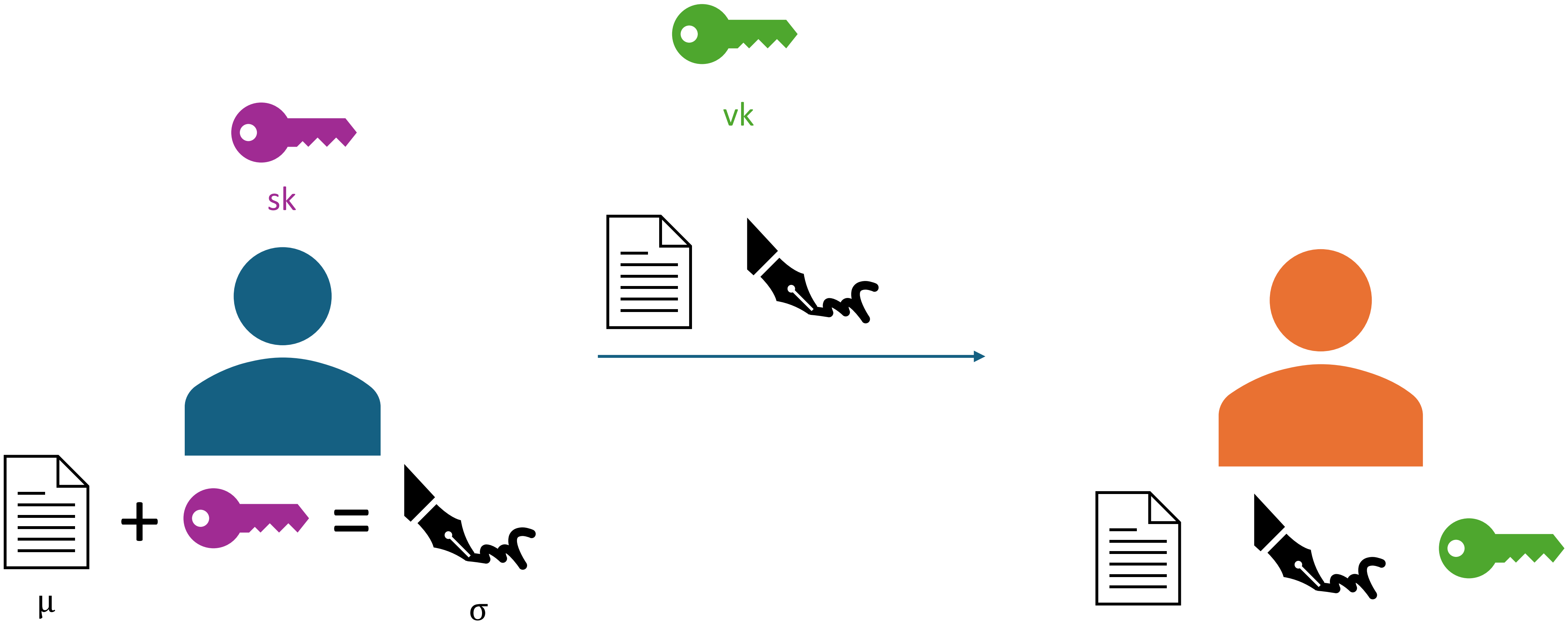

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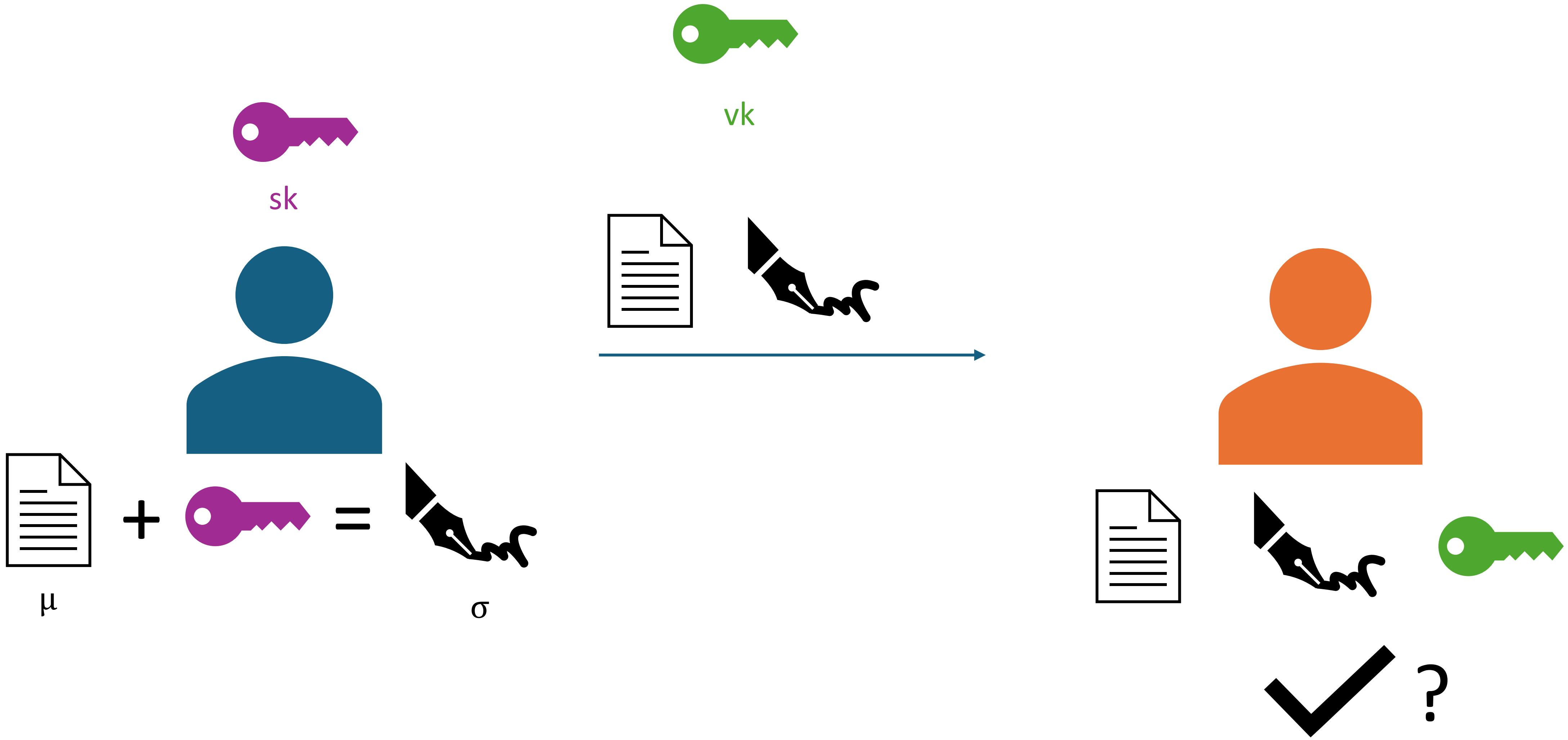

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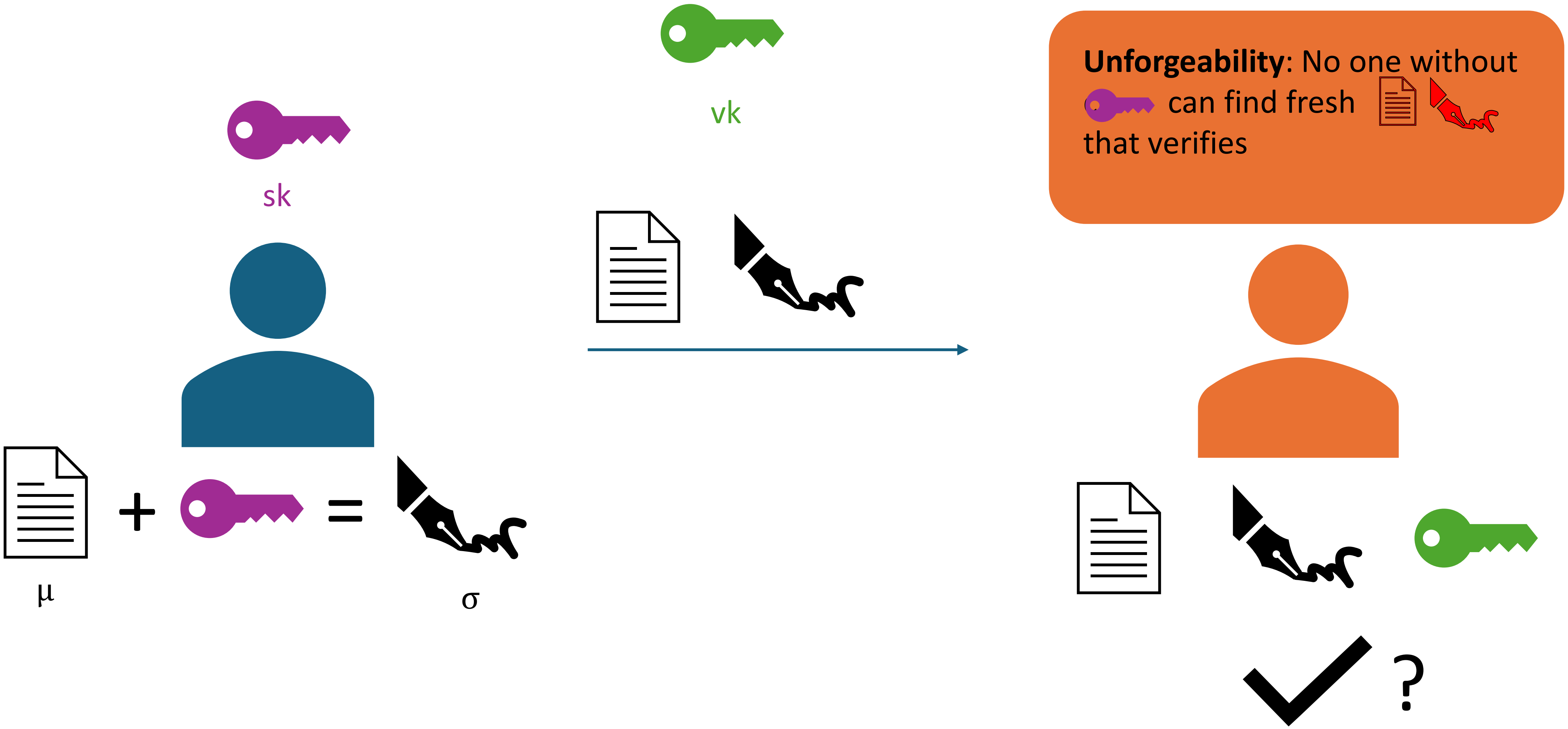

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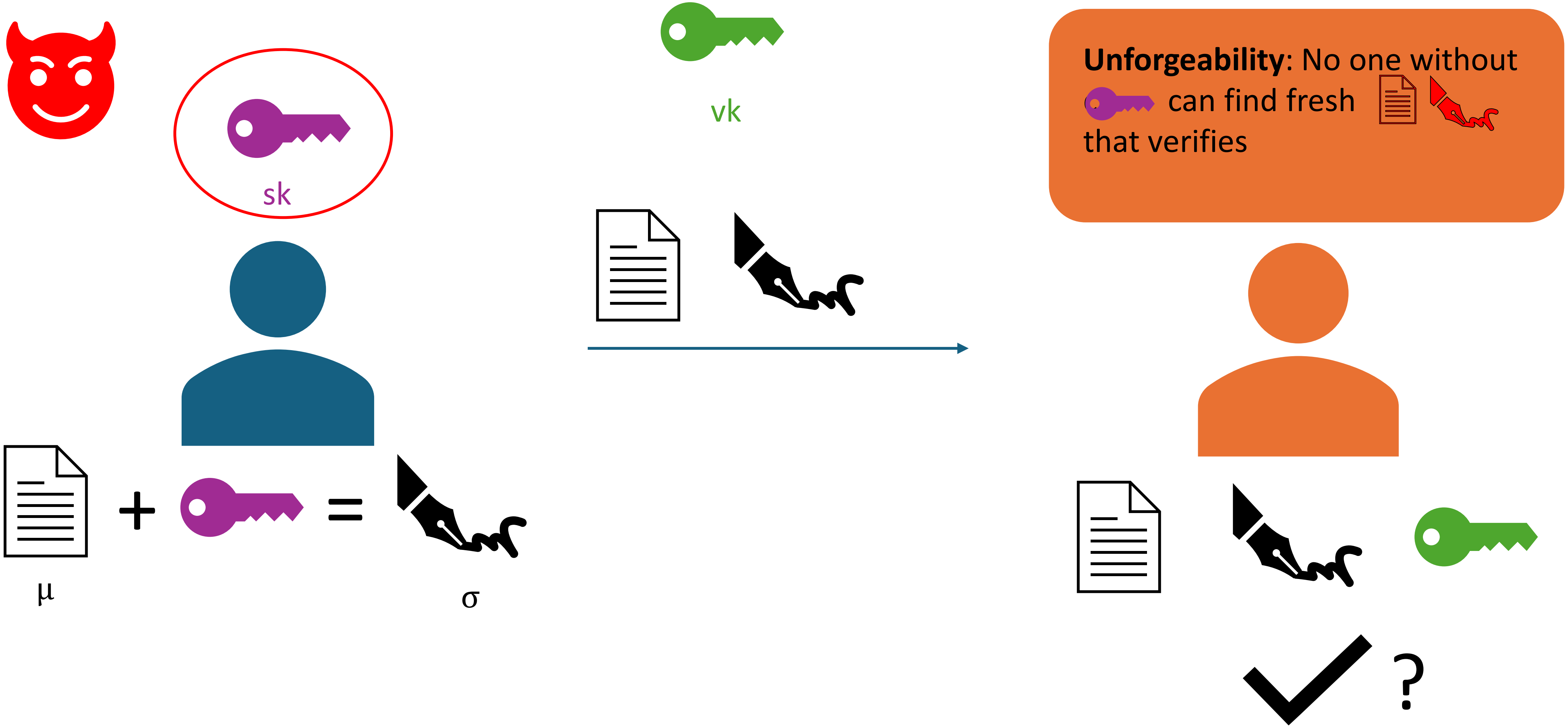

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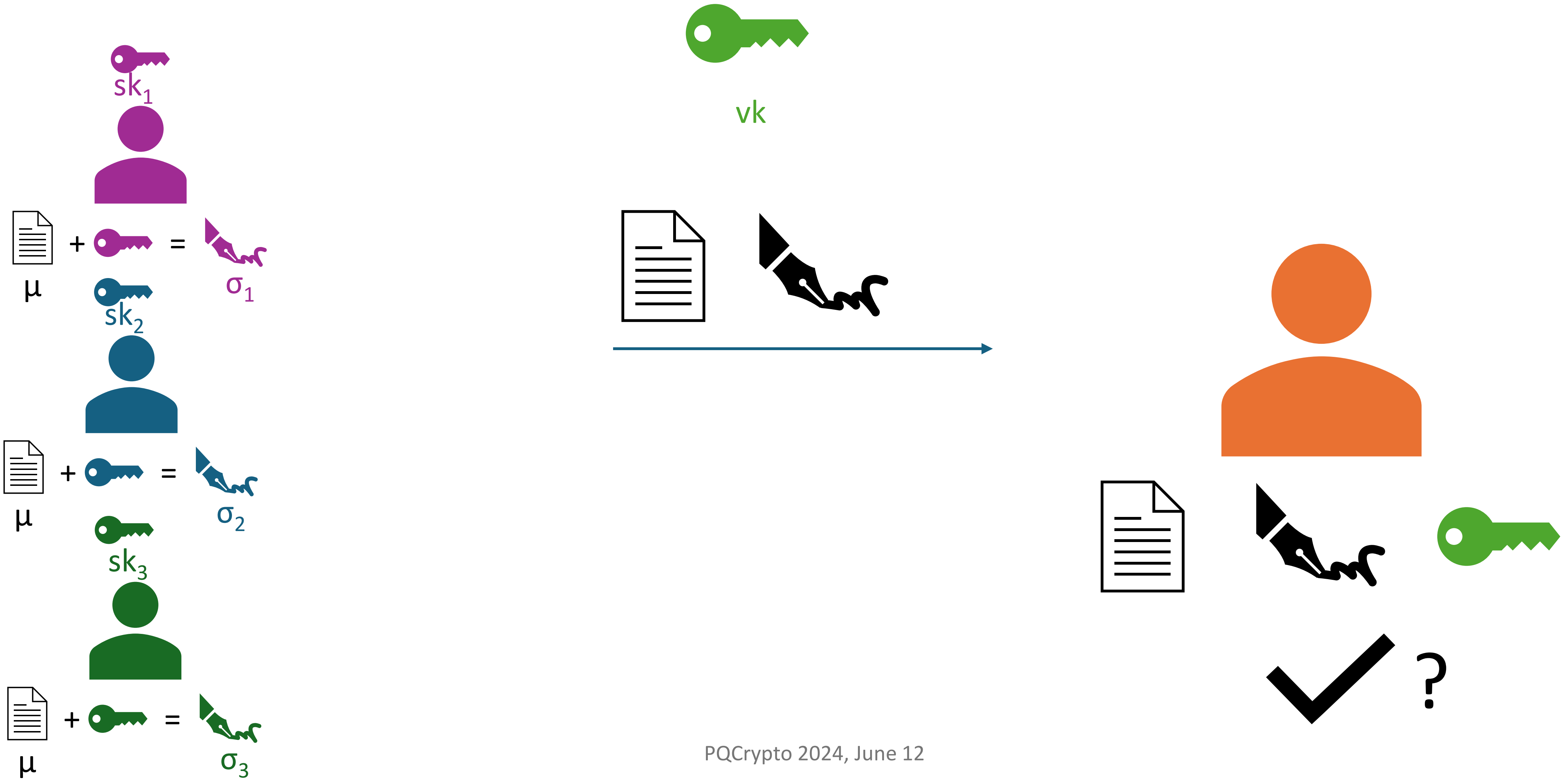

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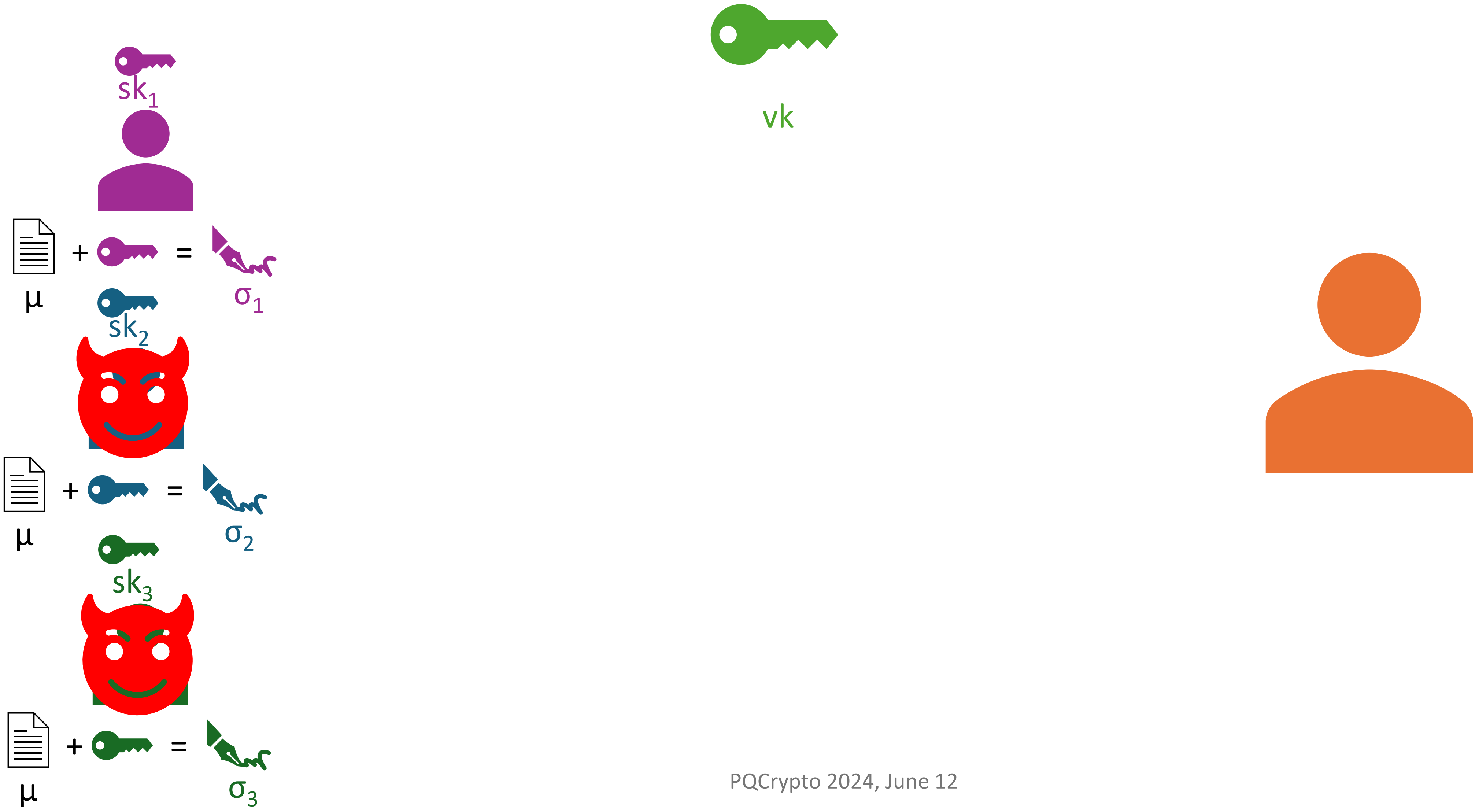

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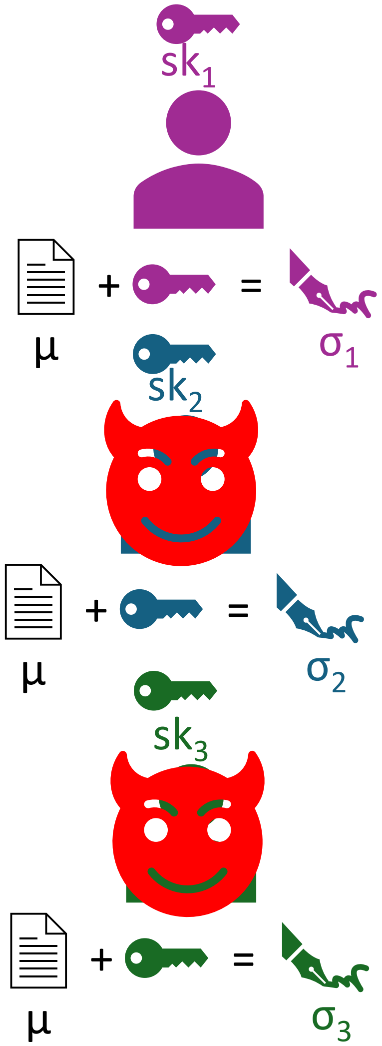

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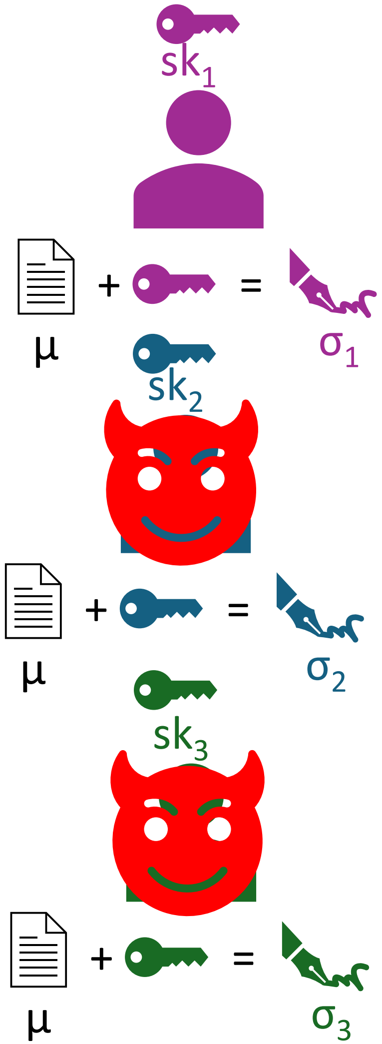

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Passive or **Active**

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  - Black-box FHE/MPC based solutions [ASY22,BGG+18,CS19]

Simple Lattice Signature Scheme

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$vk = (\bar{\boldsymbol{A}}, \boldsymbol{y})\ (\boldsymbol{y} = \bar{\boldsymbol{A}}\boldsymbol{s}, \bar{\boldsymbol{A}} = [\boldsymbol{A}|\boldsymbol{I}], \boldsymbol{A} \leftarrow R_q^{\ell \times k})$

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“rejection sampling”

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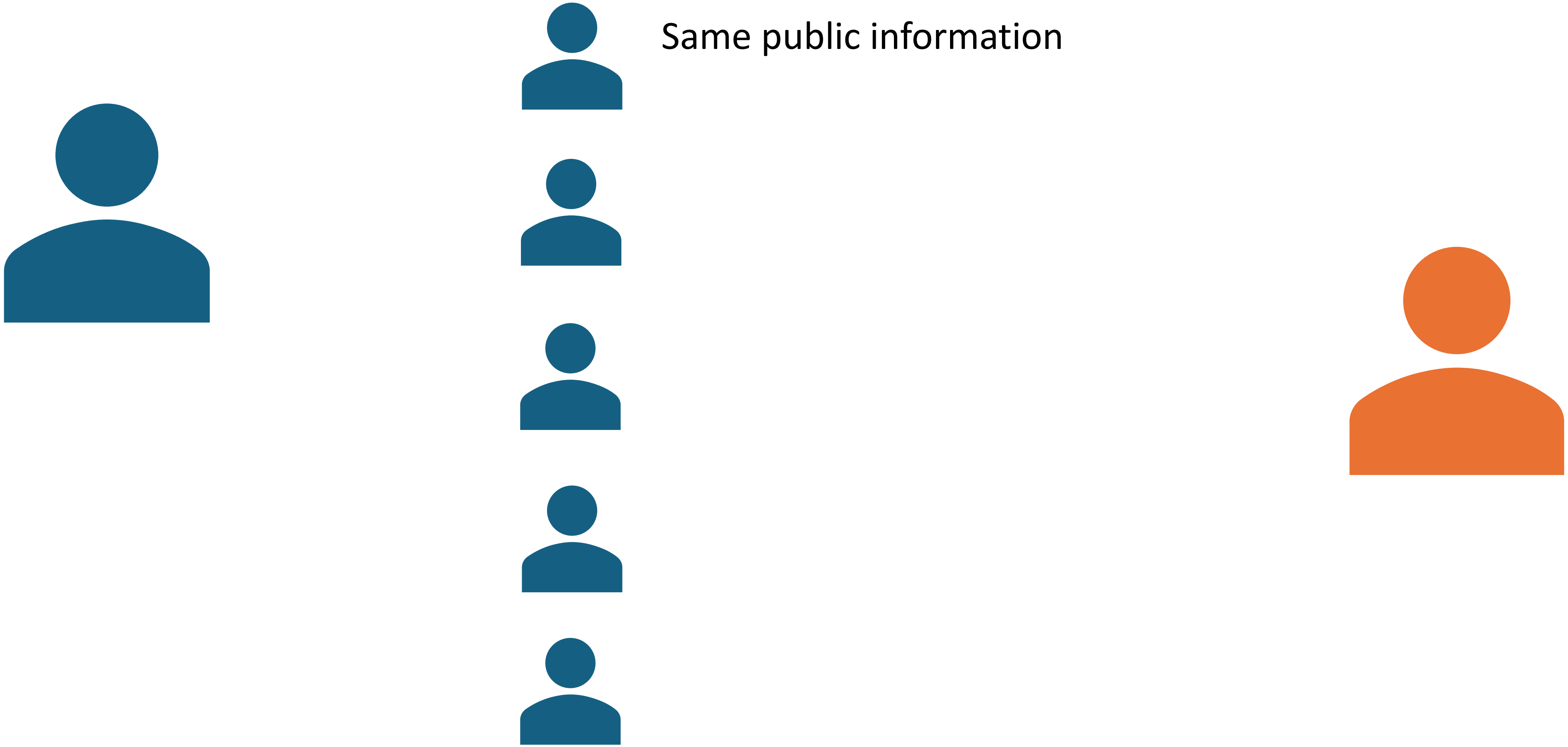

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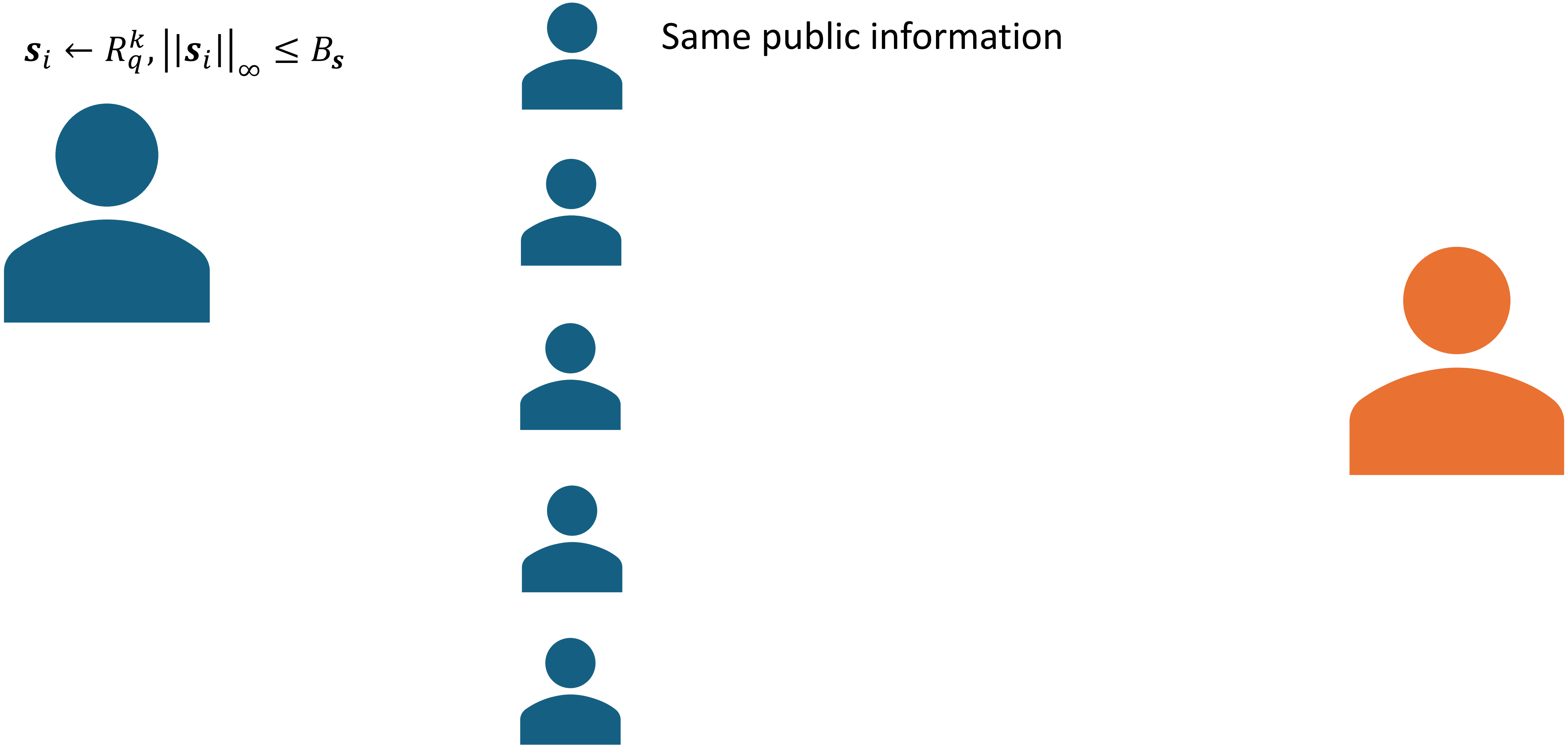

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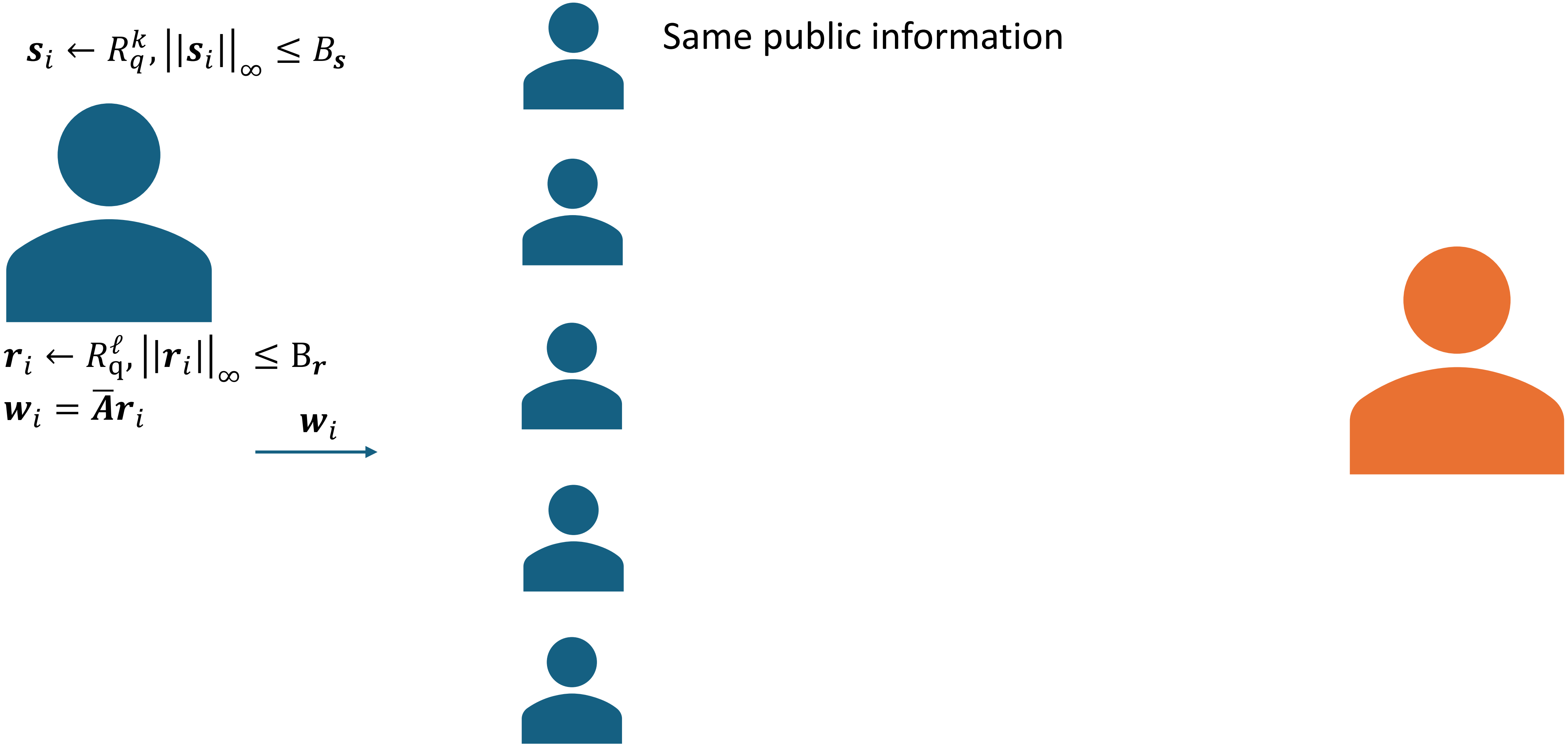

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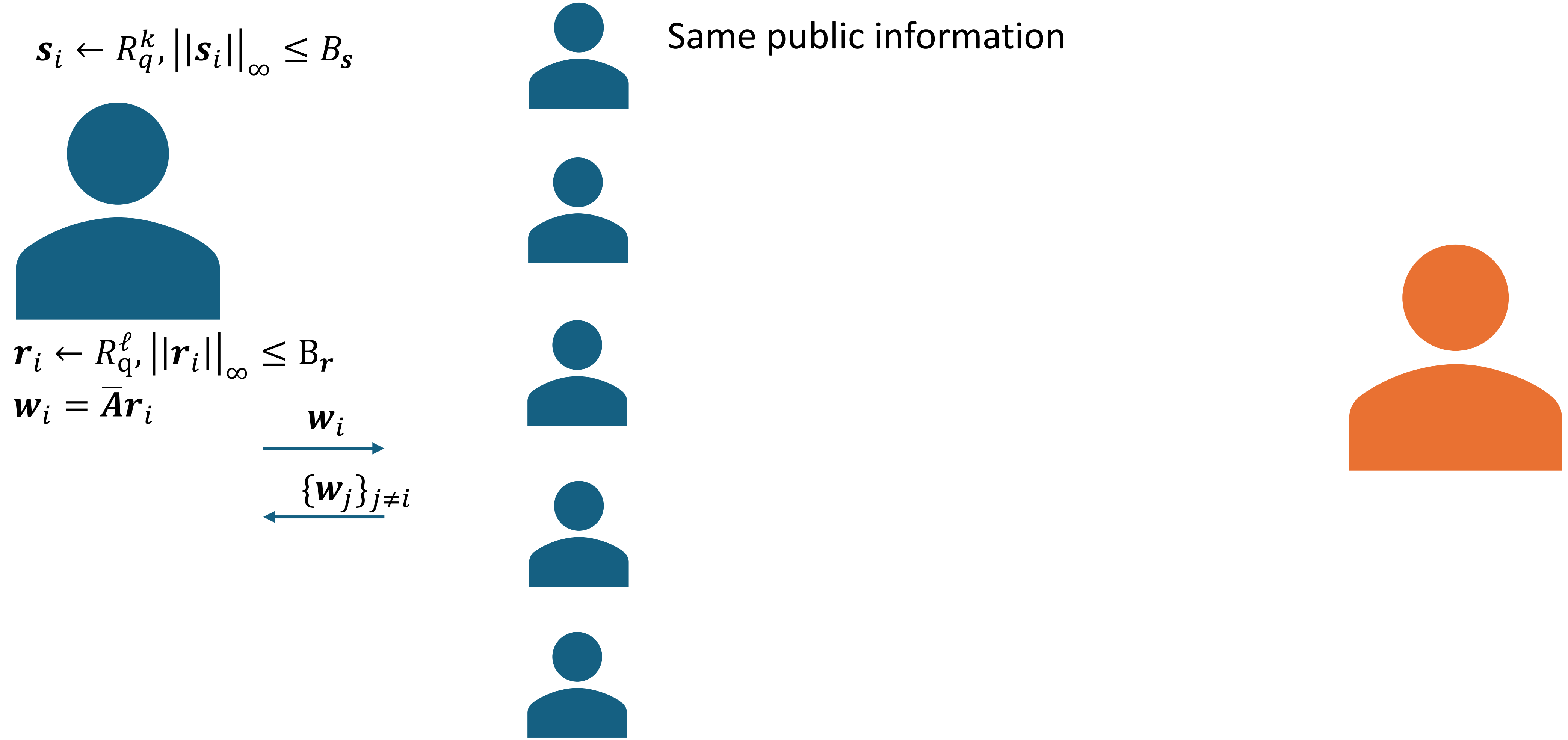

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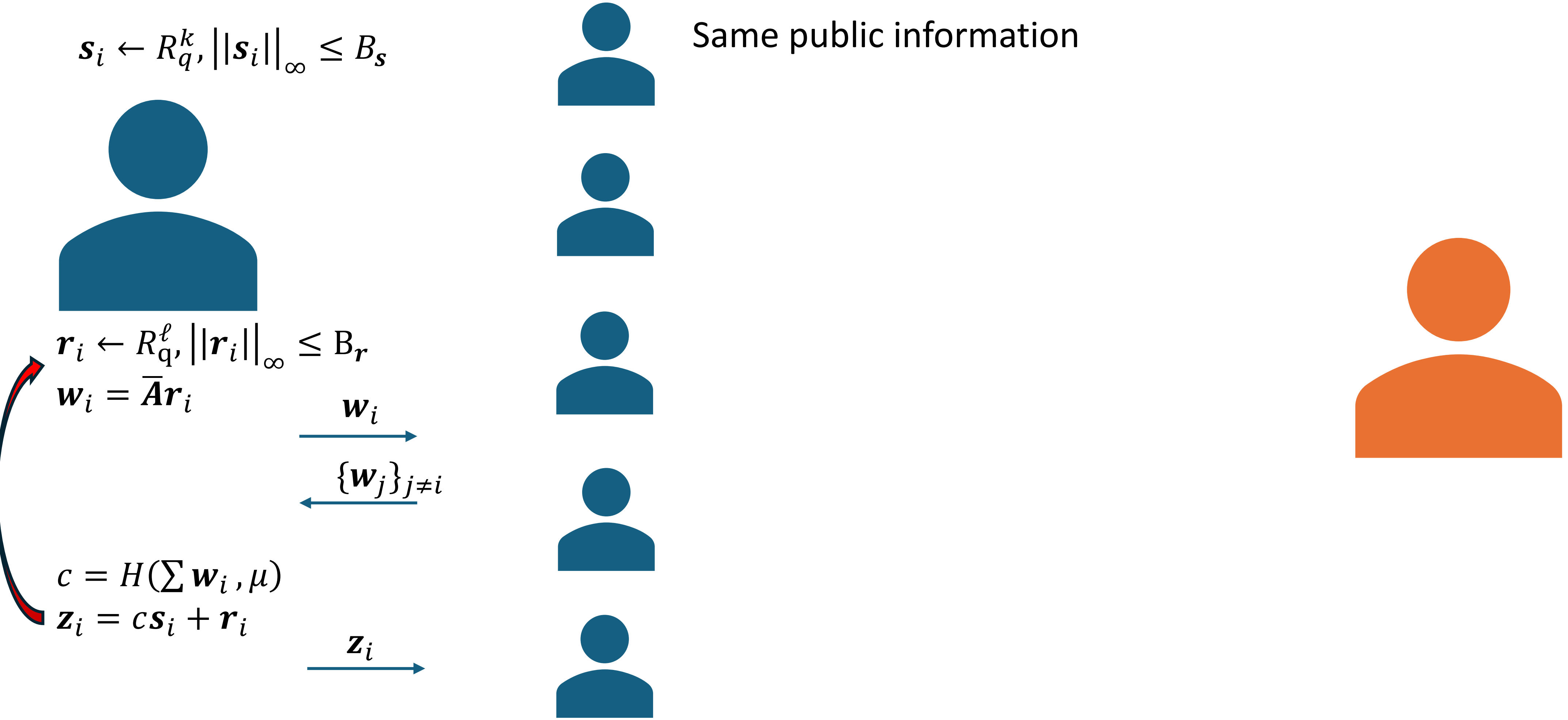

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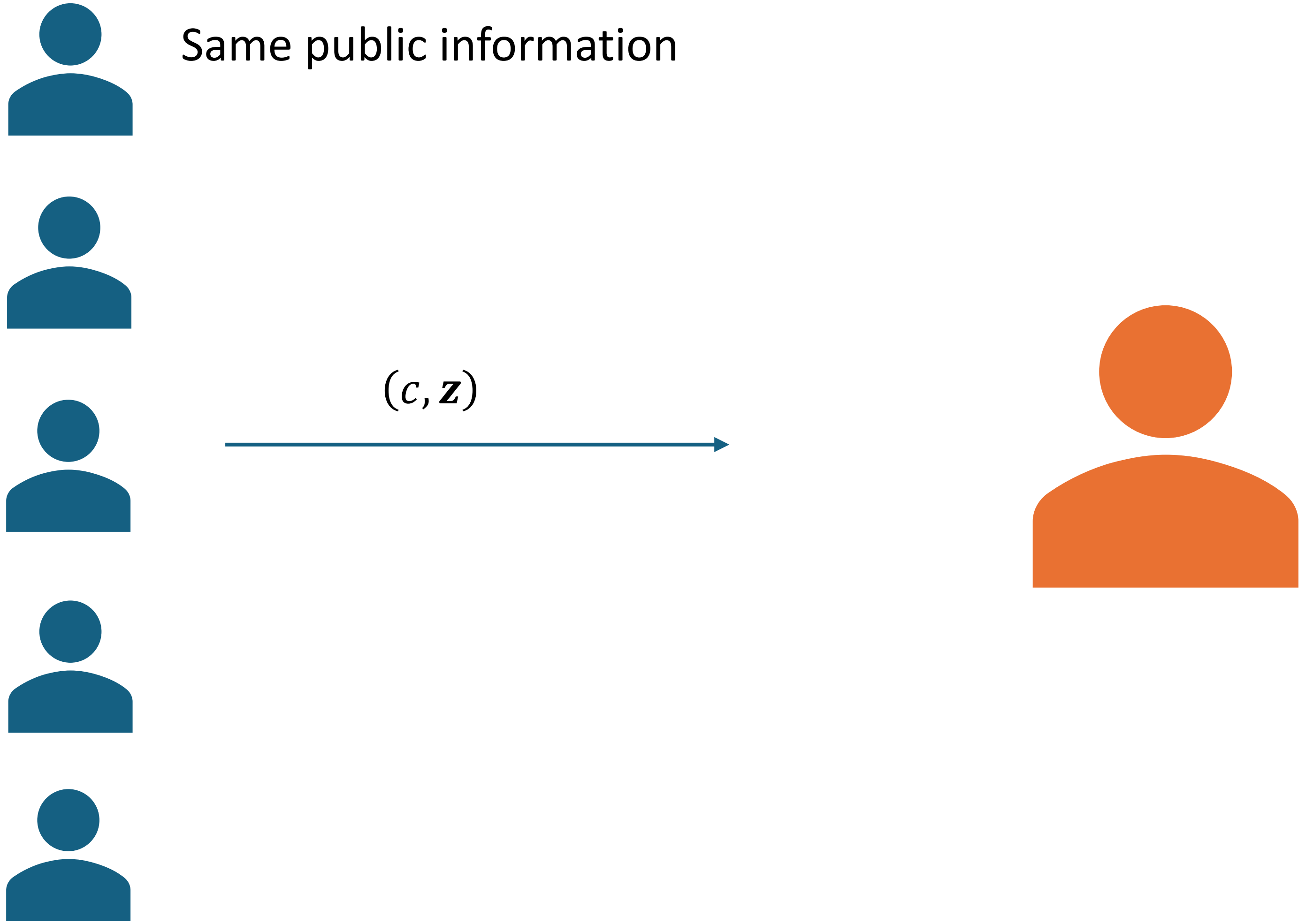

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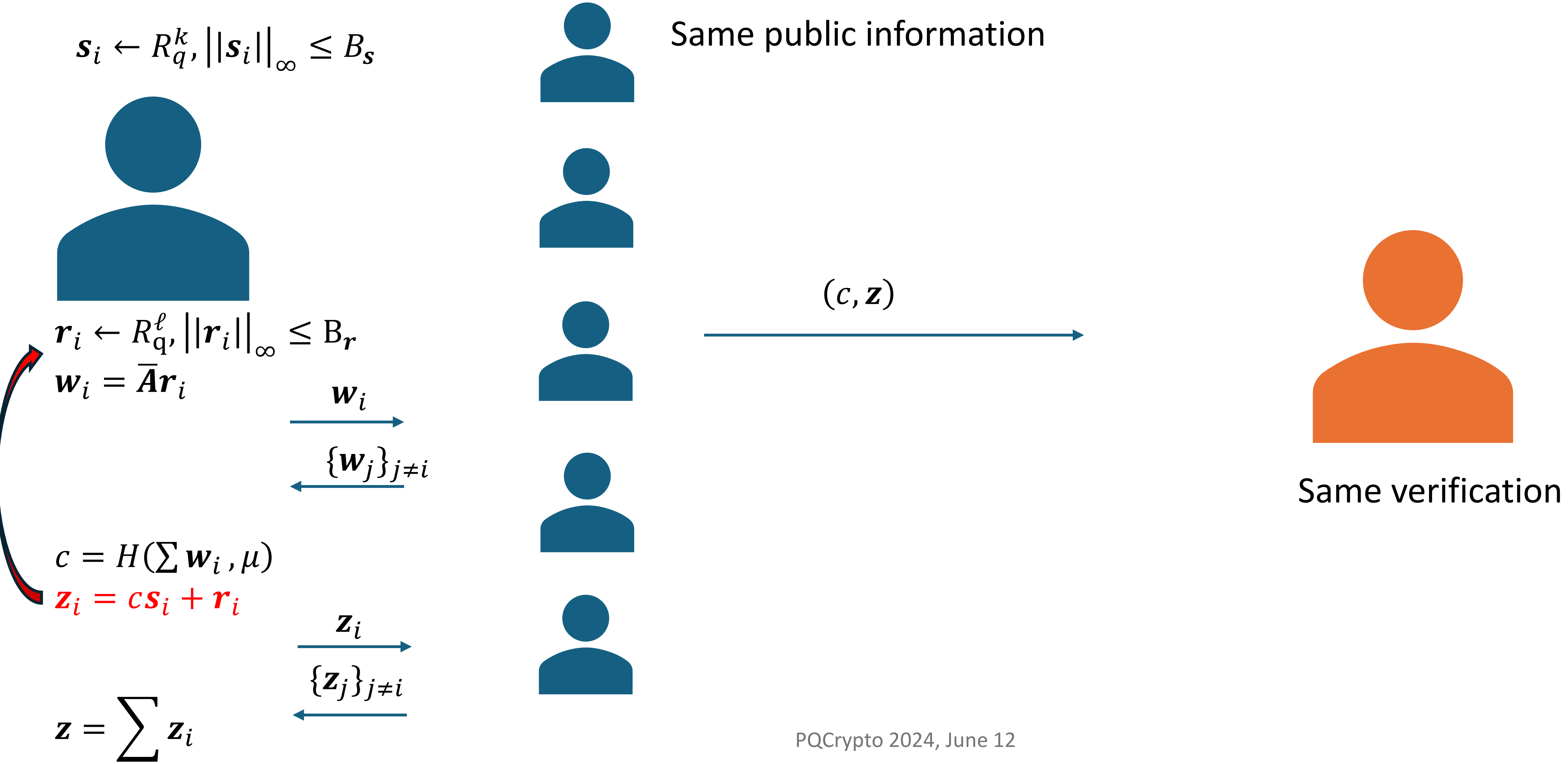

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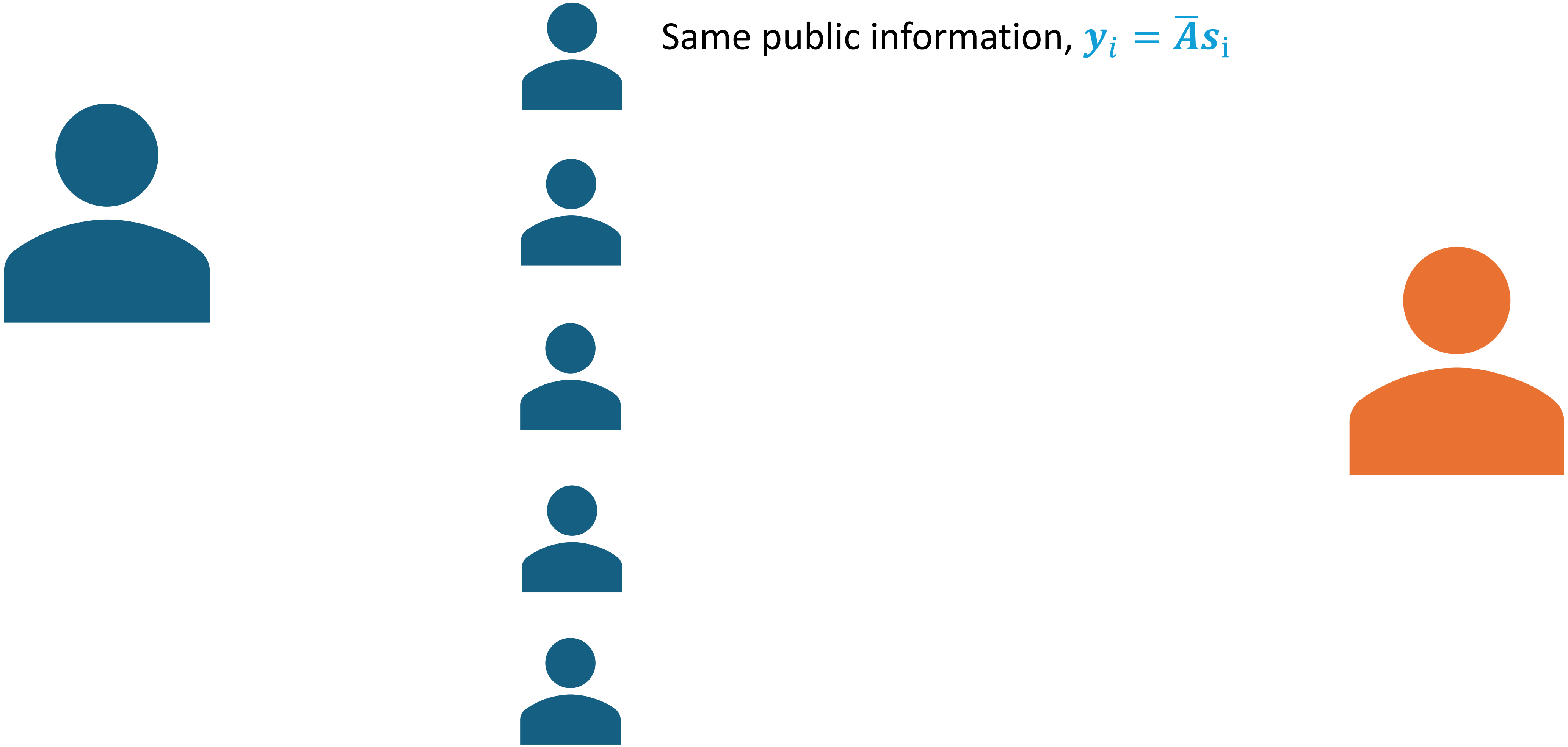

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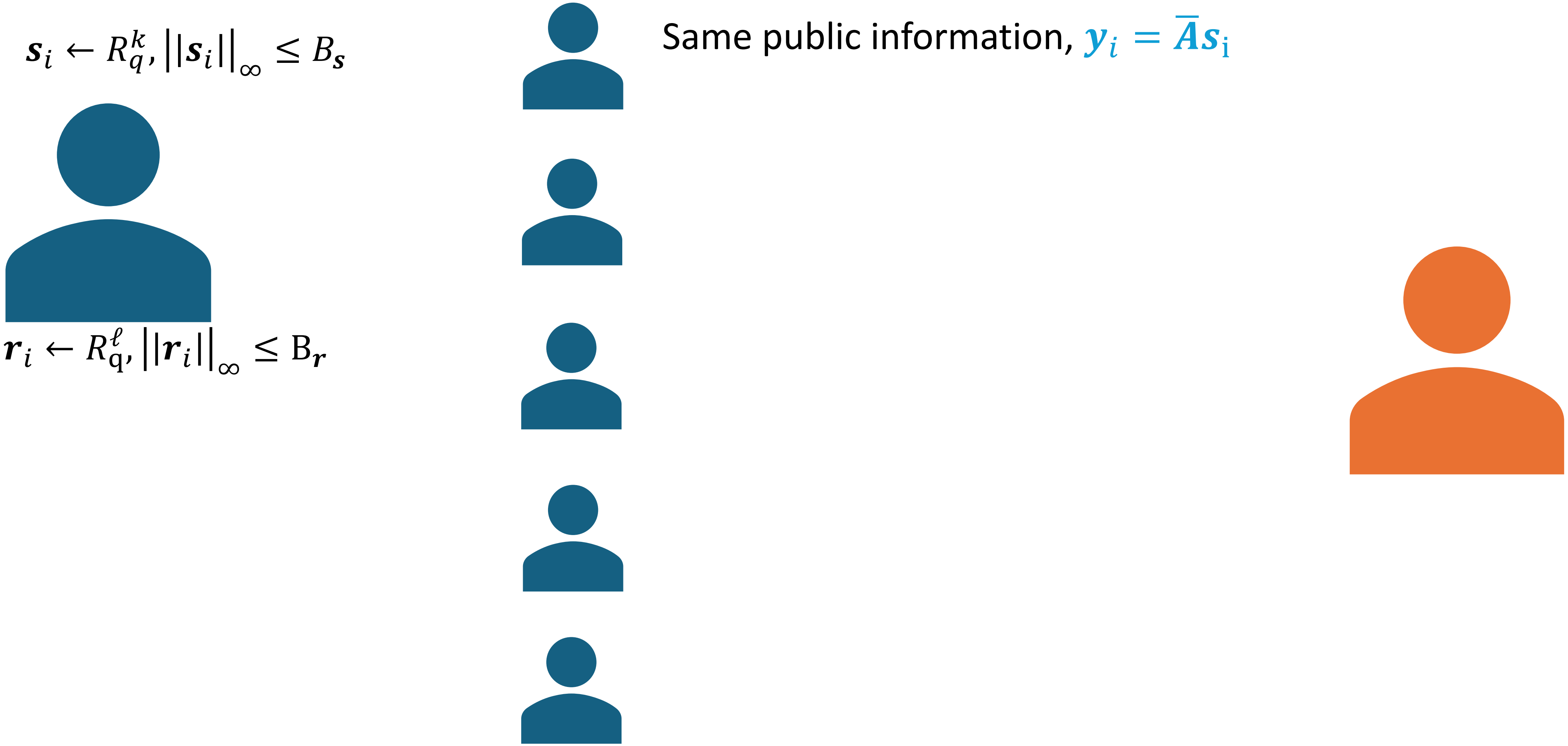

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Is there an efficient lattice-based threshold signature scheme for arbitrary thresholds?

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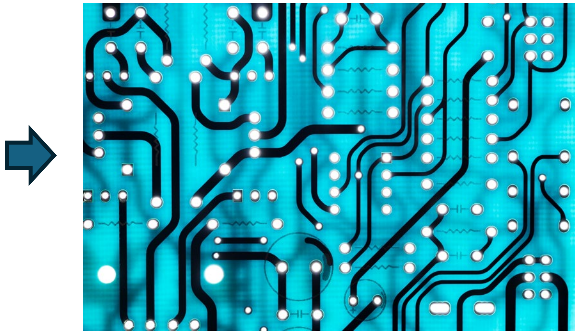

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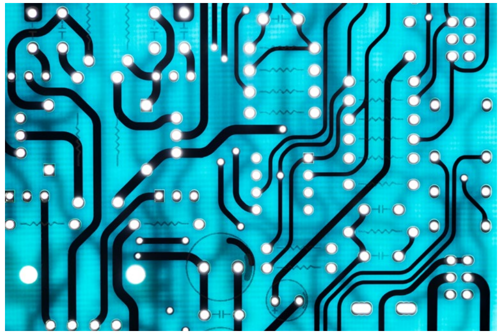

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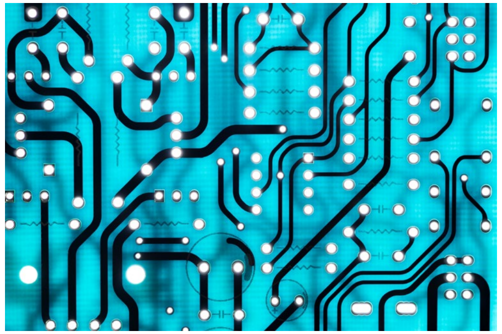

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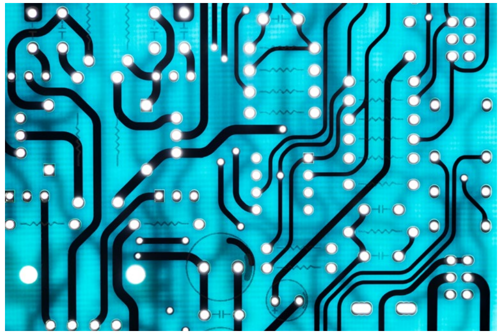

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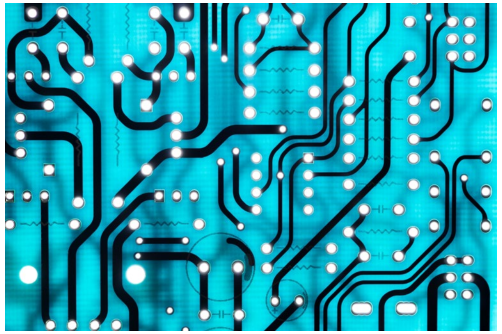

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We can use the TFHE in [BGG+18] and be done right?

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Also overlooked rejection!

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- Build a suitable threshold linearly HE
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- Combine HE with rejection-free signature

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Building Thresh

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- Weaker than standard
- Adversary allowed to be in keygen, query for partial decryptions
- **Not allowed to query on challenge ciphertext!**

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- Use <u>noise flooding</u> for decryption/signature shares

Building Threshold HE: Key Generation

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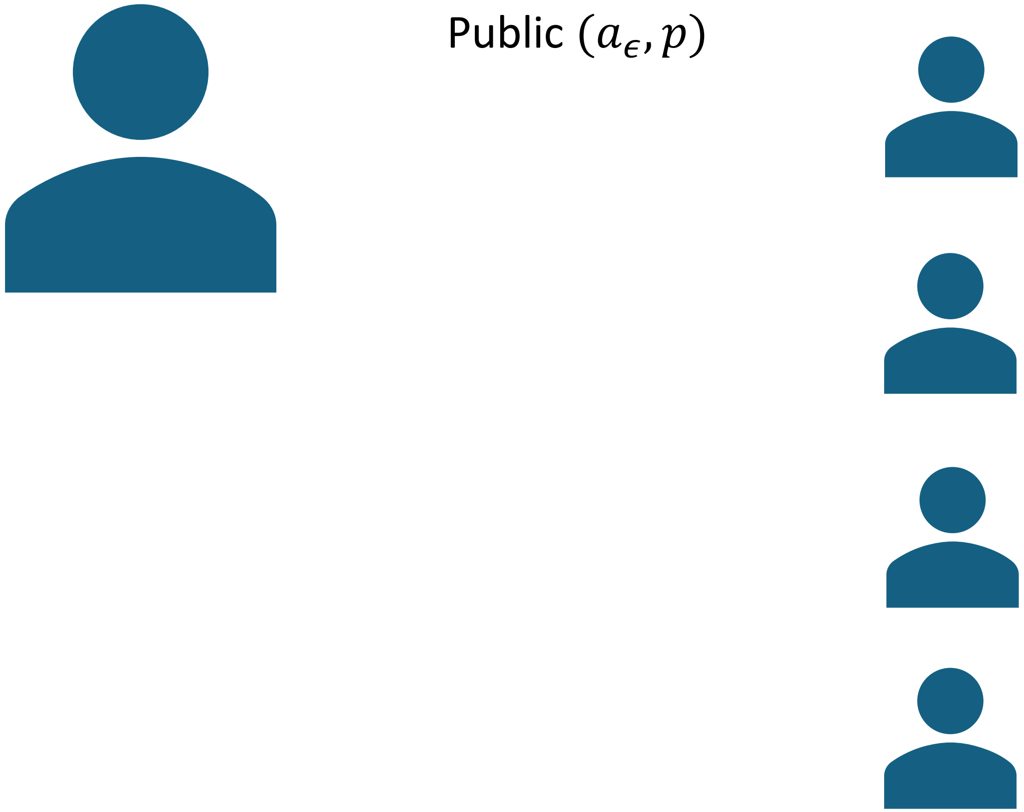

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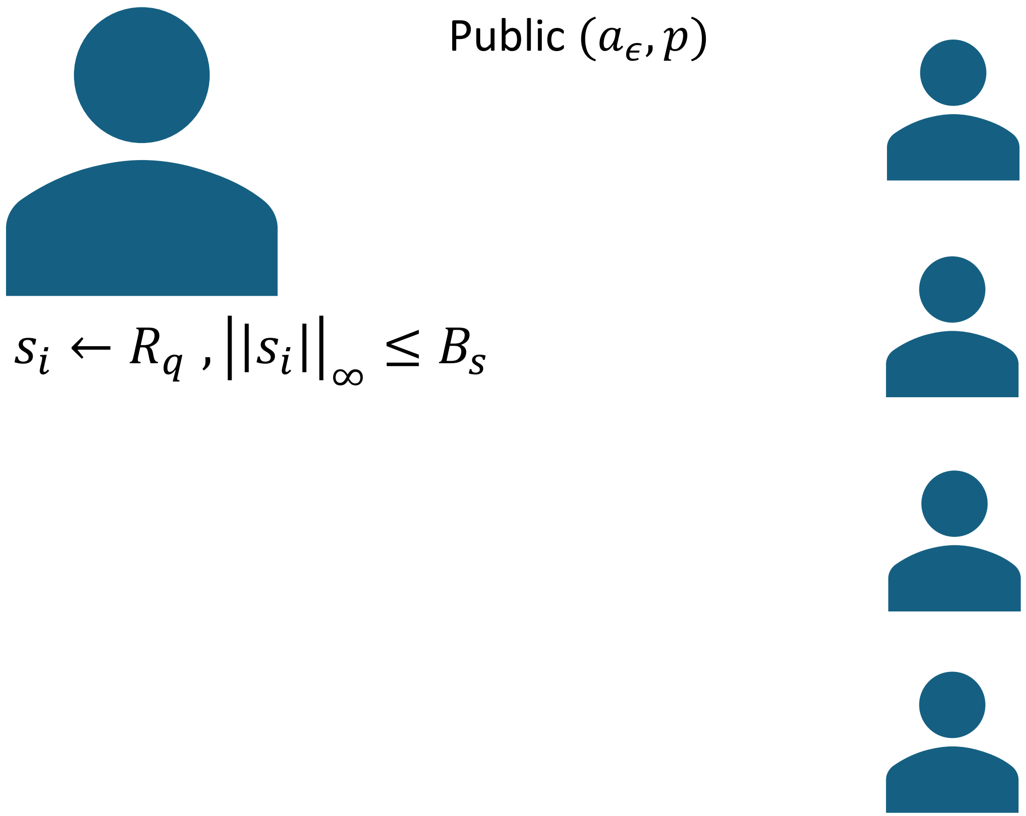

Building Threshold HE: Key Generation

Public (a_ϵ, p)

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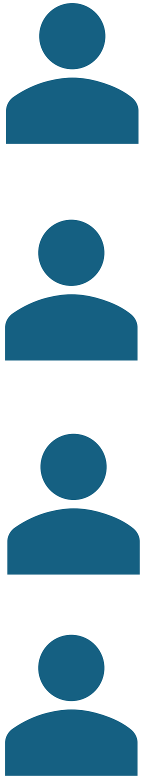
$e_i \leftarrow D_e, b_i = a\ s_i + p\ e_i$



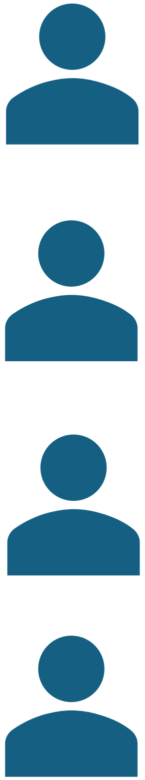
Building Threshold HE: Key Generation

$s_i \leftarrow R_q$, $||s_i||_\infty \leq B_s$

$e_i \leftarrow D_e, b_i = a\ s_i + p\ e_i$

(t, n) secret share s_i, e_i

Public (a_ϵ, p)



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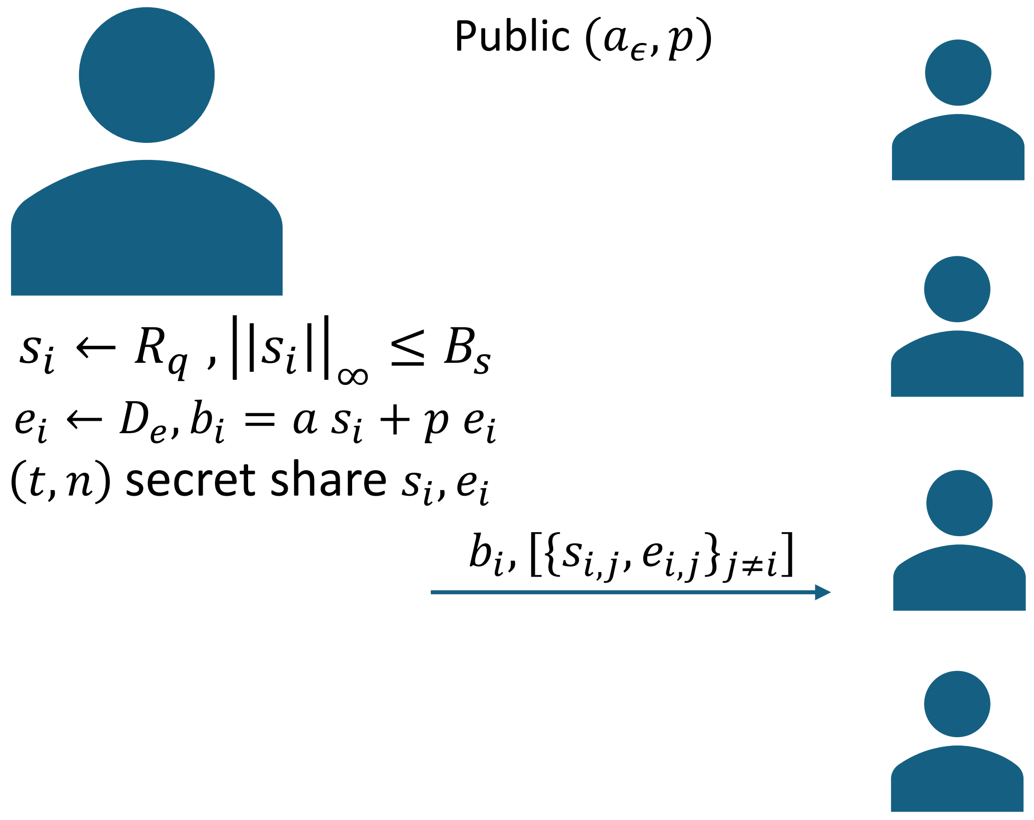
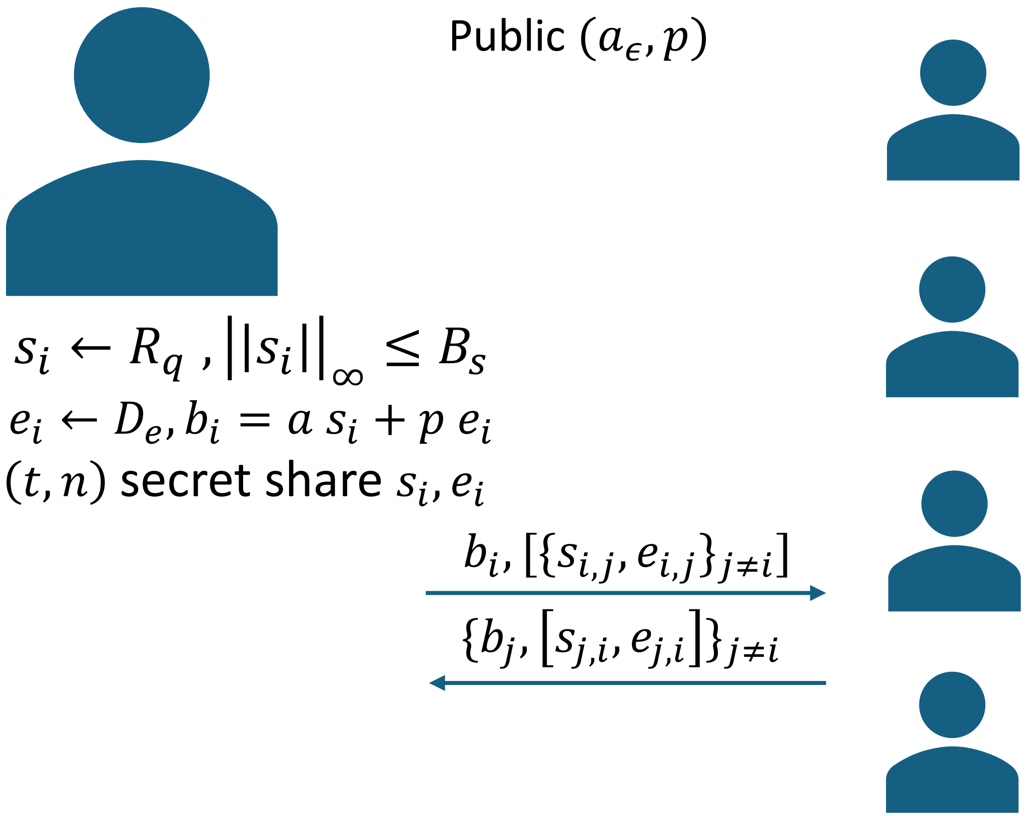


Building Threshold HE: Key Generation



Building Threshold HE: Key Generation

Public (a_ϵ, p)

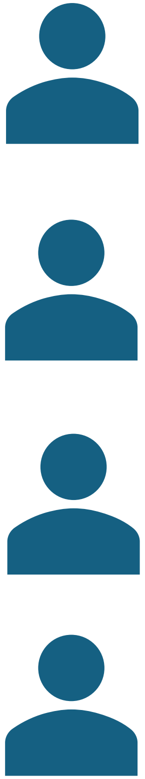
$s_i \leftarrow R_q \,, \left|\left|s_i\right|\right|_\infty \leq B_s$

$e_i \leftarrow D_e, b_i = a\, s_i + p\, e_i$

(t, n) secret share s_i, e_i

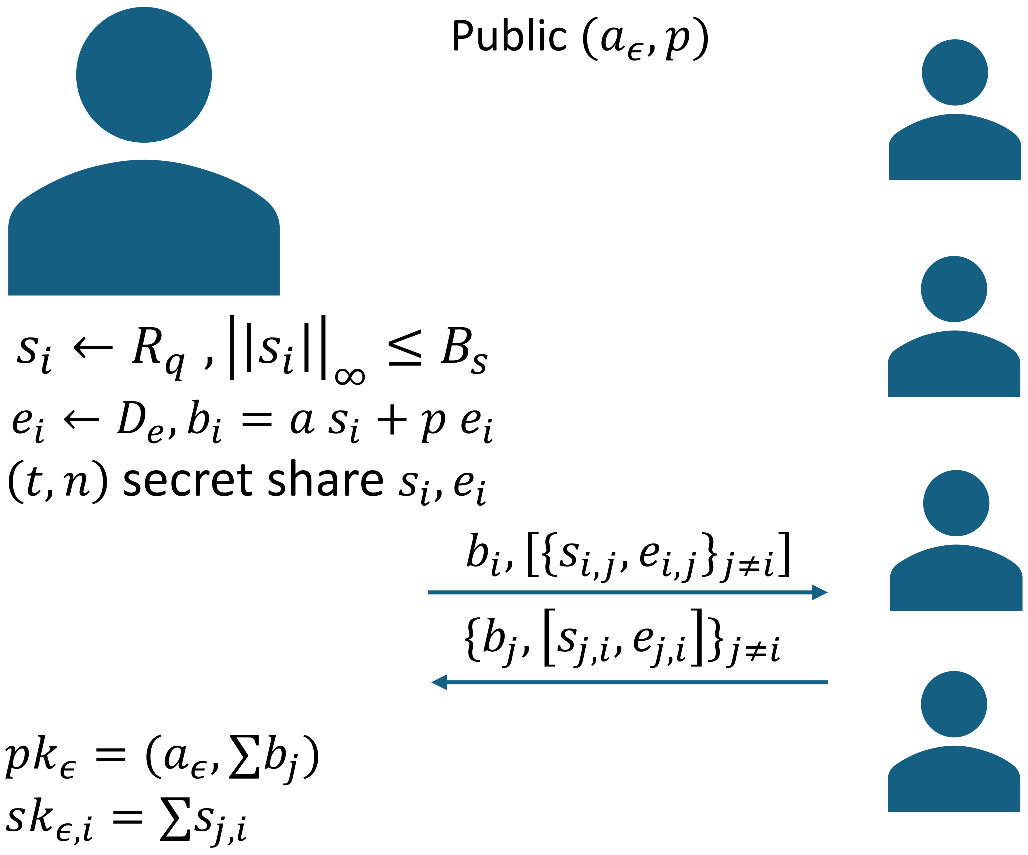
$b_i, [\{s_{i,j}, e_{i,j}\}_{j \neq i}]$ →

← $\{b_j, [s_{j,i}, e_{j,i}]\}_{j \neq i}$



$pk_\epsilon = (a_\epsilon, \sum b_j)$

Building Threshold HE: Key Generation



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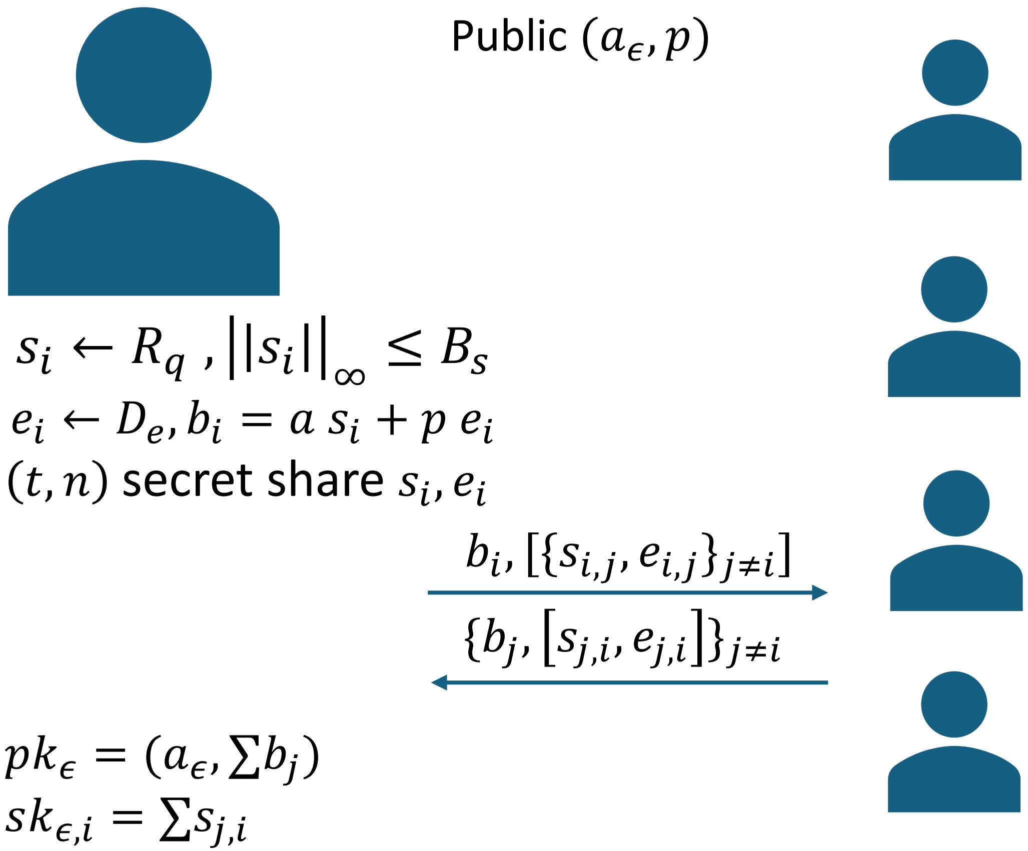
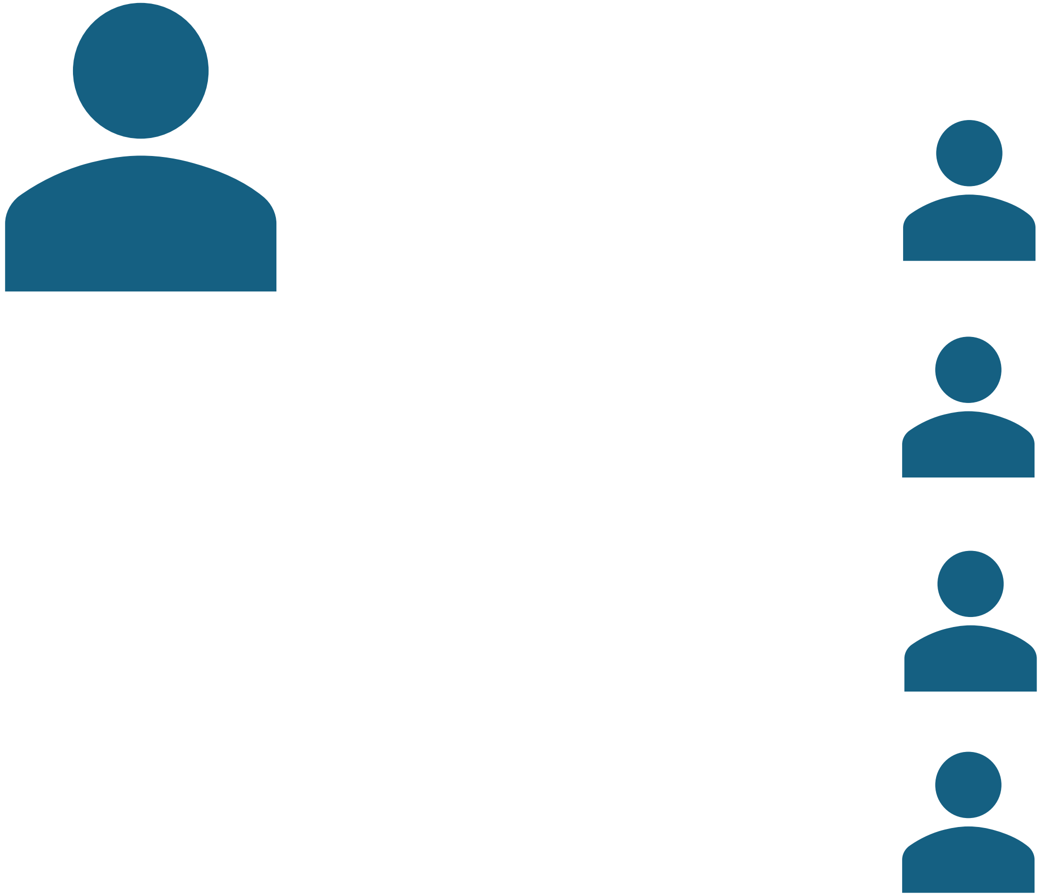


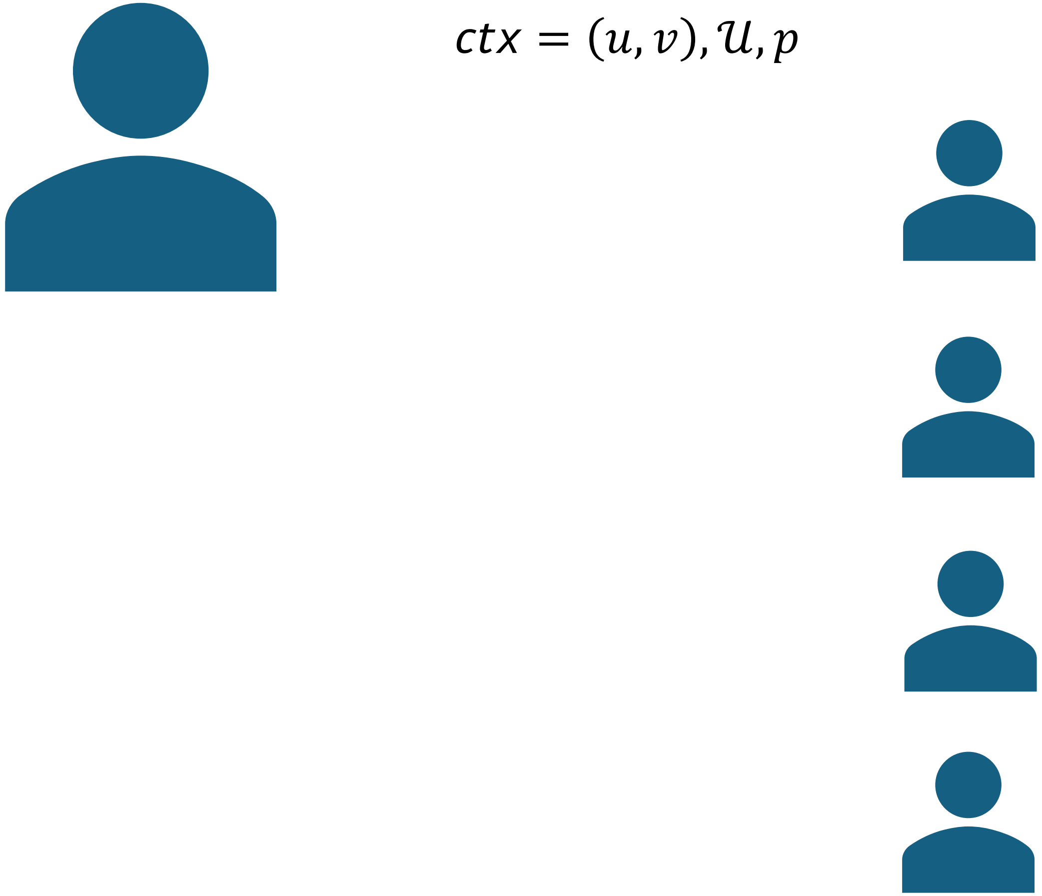
Commitments and ZKPs for active security

Building Threshold HE: Threshold Decryption



Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$



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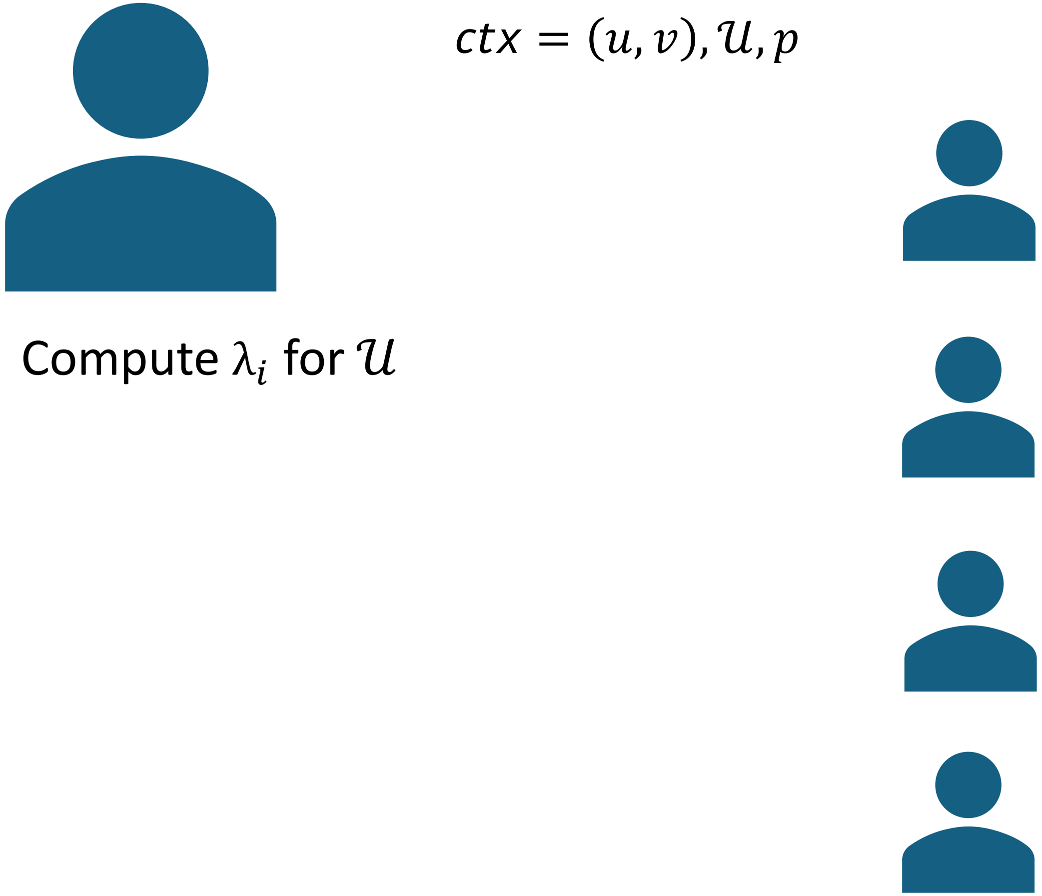
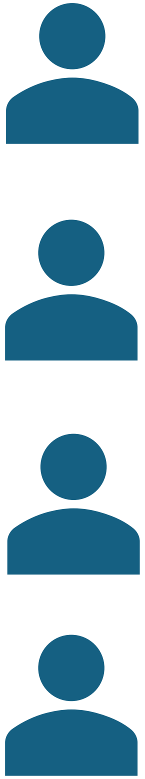


Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$

Compute λ_i for $\mathcal{U}$

$E_i \leftarrow D_{TDec}$



Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$

Compute λ_i for $\mathcal{U}$

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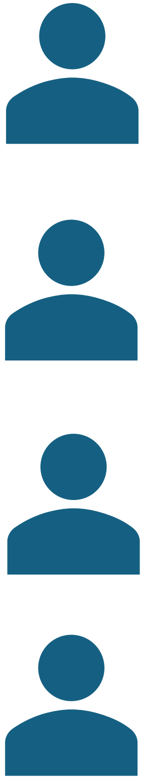
$d_i = \lambda_i sk_{\epsilon,i} u + pE_i$



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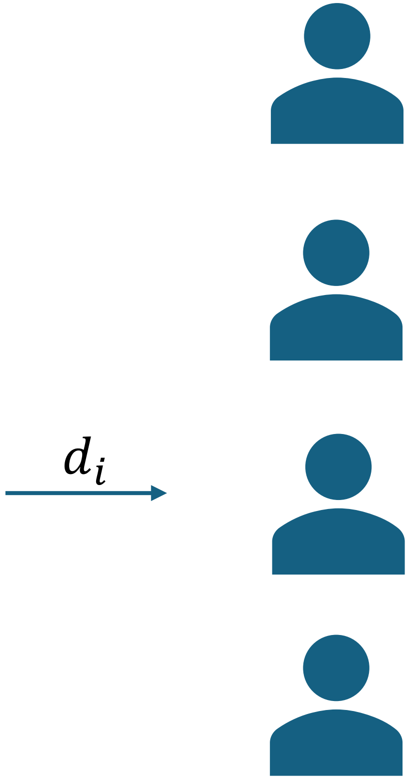
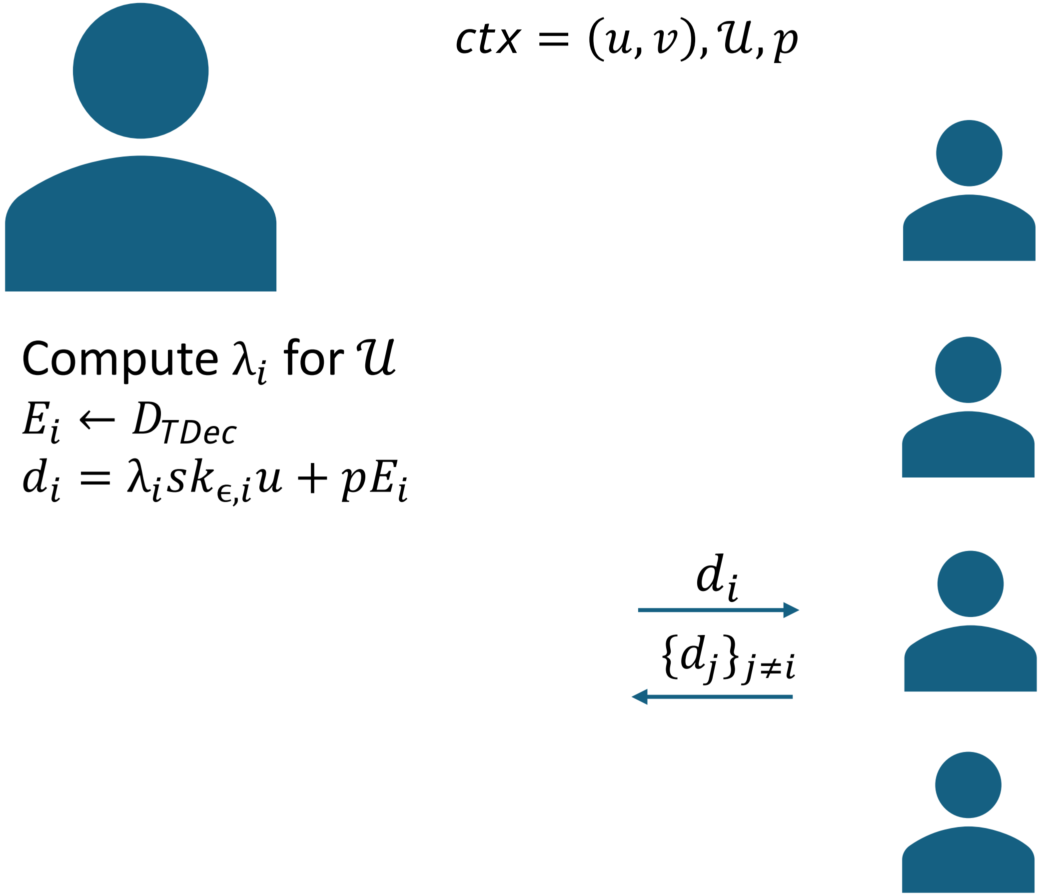


Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$



Compute λ_i for $\mathcal{U}$

$E_i \leftarrow D_{TDec}$

$d_i = \lambda_i sk_{\epsilon,i} u + pE_i$

d_i

$\{d_j\}_{j \neq i}$

Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$

Compute λ_i for $\mathcal{U}$

$E_i \leftarrow D_{TDec}$

$d_i = \lambda_i sk_{\epsilon,i} u + pE_i$

d_i →

← $\{d_j\}_{j \neq i}$

$ptx = v - \sum d_j \bmod p$

Building Threshold HE: Threshold Decryption

$ctx = (u, v), \mathcal{U}, p$

Compute λ_i for $\mathcal{U}$

$E_i \leftarrow D_{TDec}$

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Commitments and ZKPs for active security

Organization

- Build a suitable threshold HE
- Avoid rejection sampling
- Combine HE with rejection-free signature

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- Rejection prevents leaking **s** with each query

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- Just remove it!

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Avoid rejection sampling

- Rejection prevents leaking **s** with each query
- Just remove it!
  - Use the same **s** for a bounded number of queries
  - Analysis based on Renyi divergence(RD) [BLR+18]
  - [ASY22] and “gentle noise flooding”

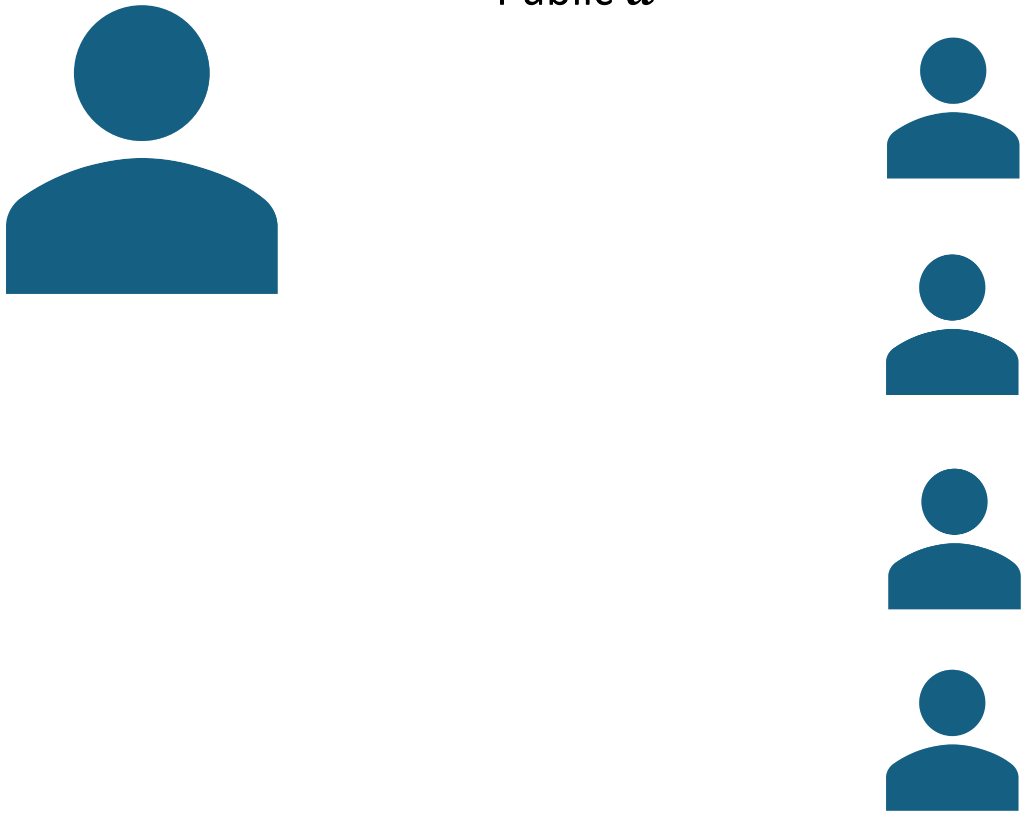
Organization

- Build a suitable threshold HE
- Deal with rejection sampling
- Combine HE with rejection-free signature

Building Signatures: Key Generation

Building Signatures: Key Generation

Public $\boldsymbol{a}$



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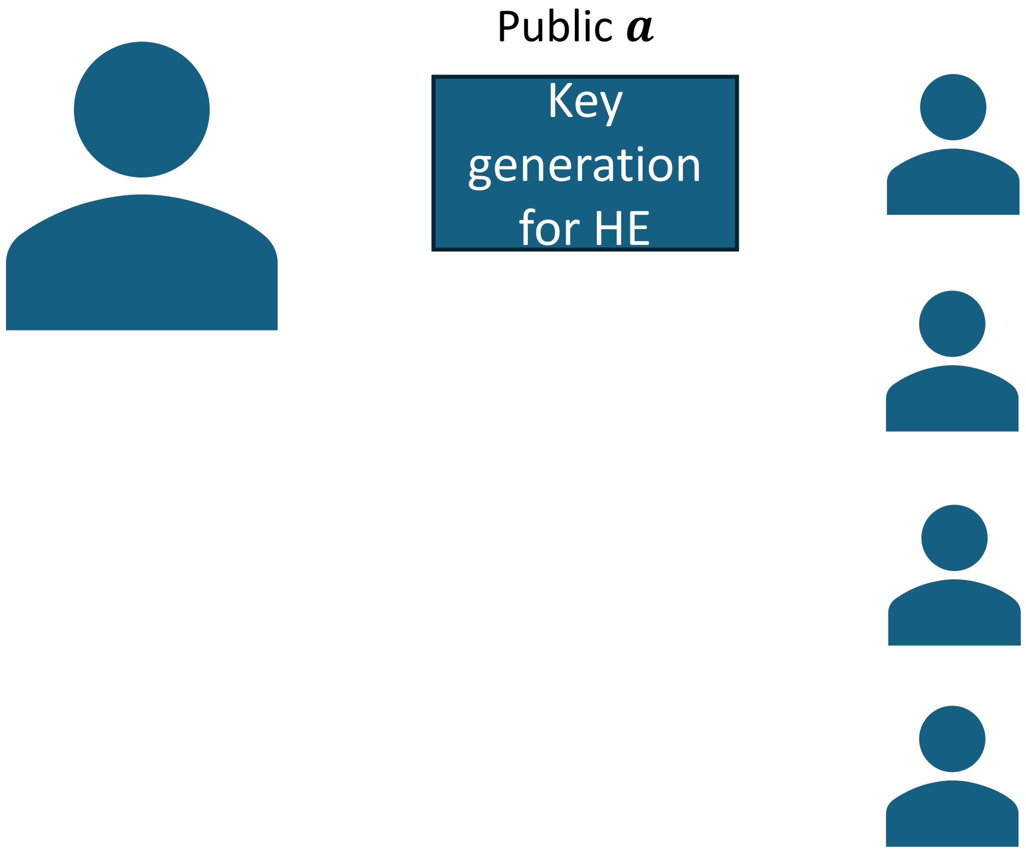

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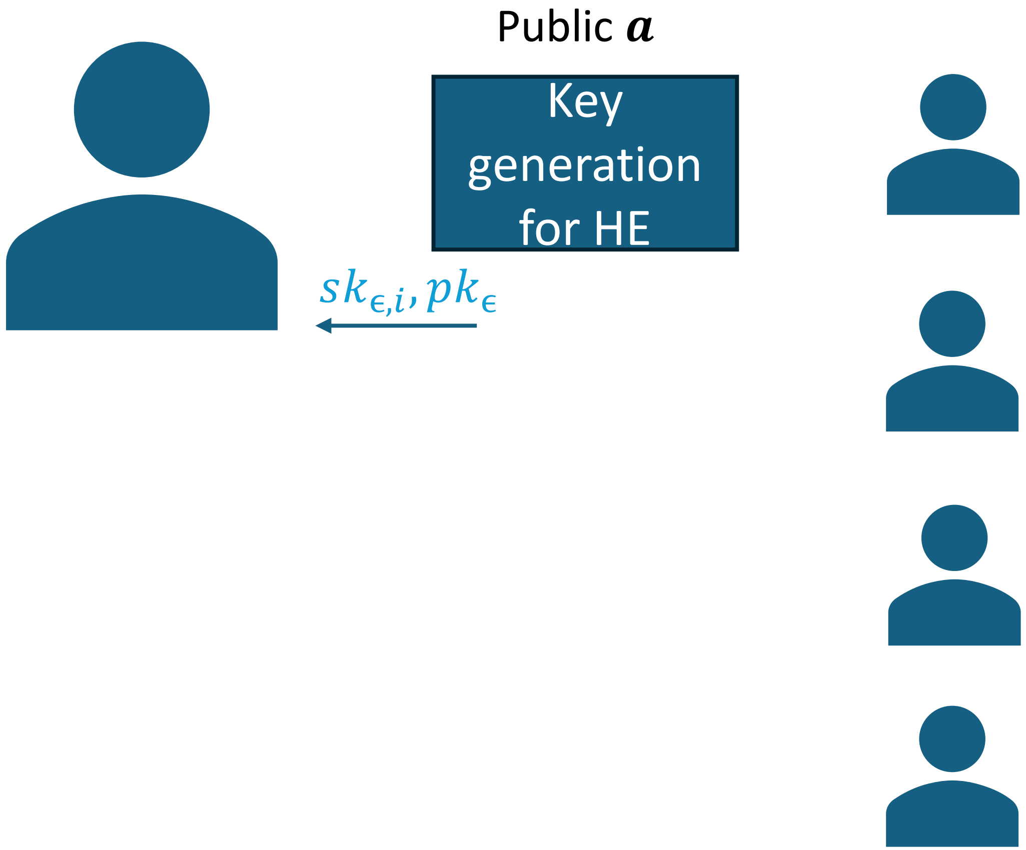
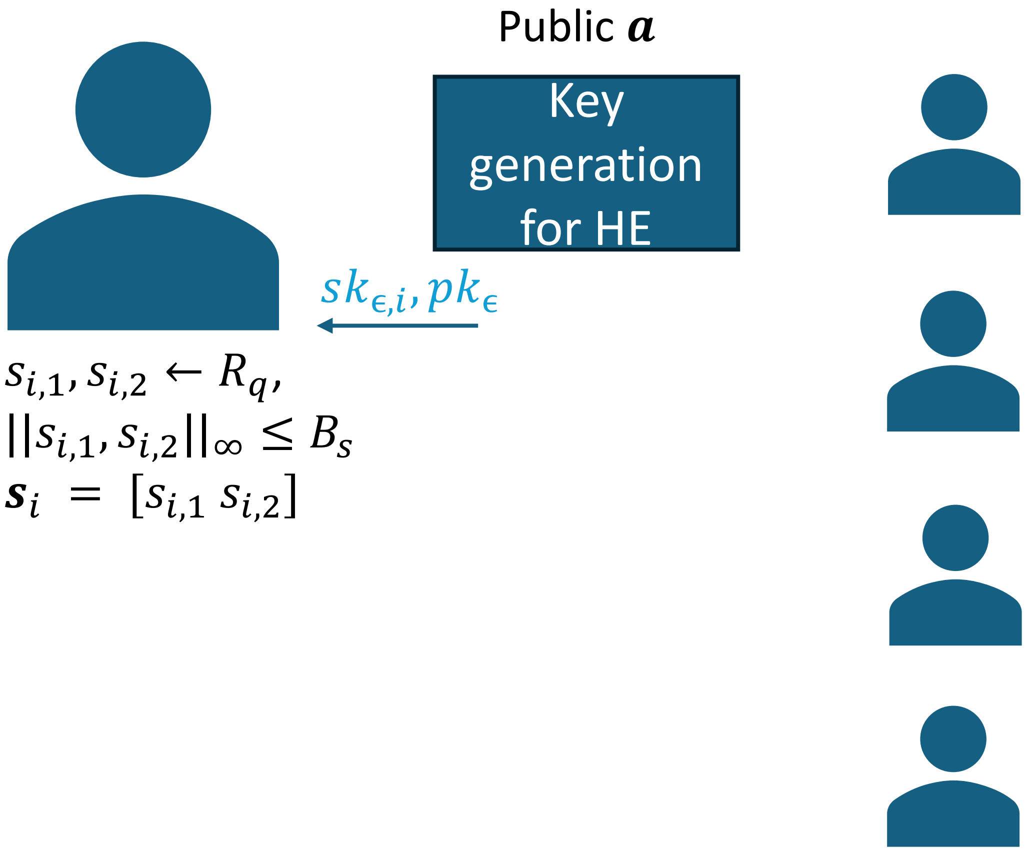


Building Signatures: Key Generation



$s_{i,1}, s_{i,2} \leftarrow R_q,$
$||s_{i,1}, s_{i,2}||_\infty \leq B_s$
$\boldsymbol{s}_i = [s_{i,1}\ s_{i,2}]$

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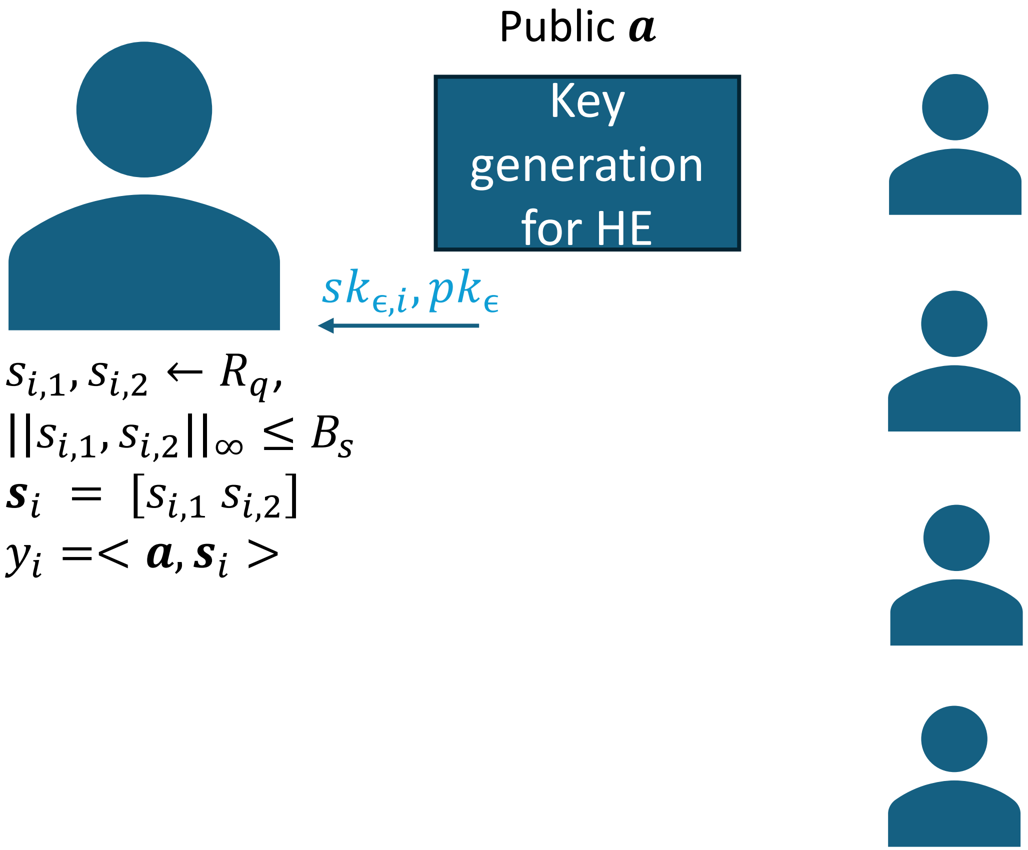

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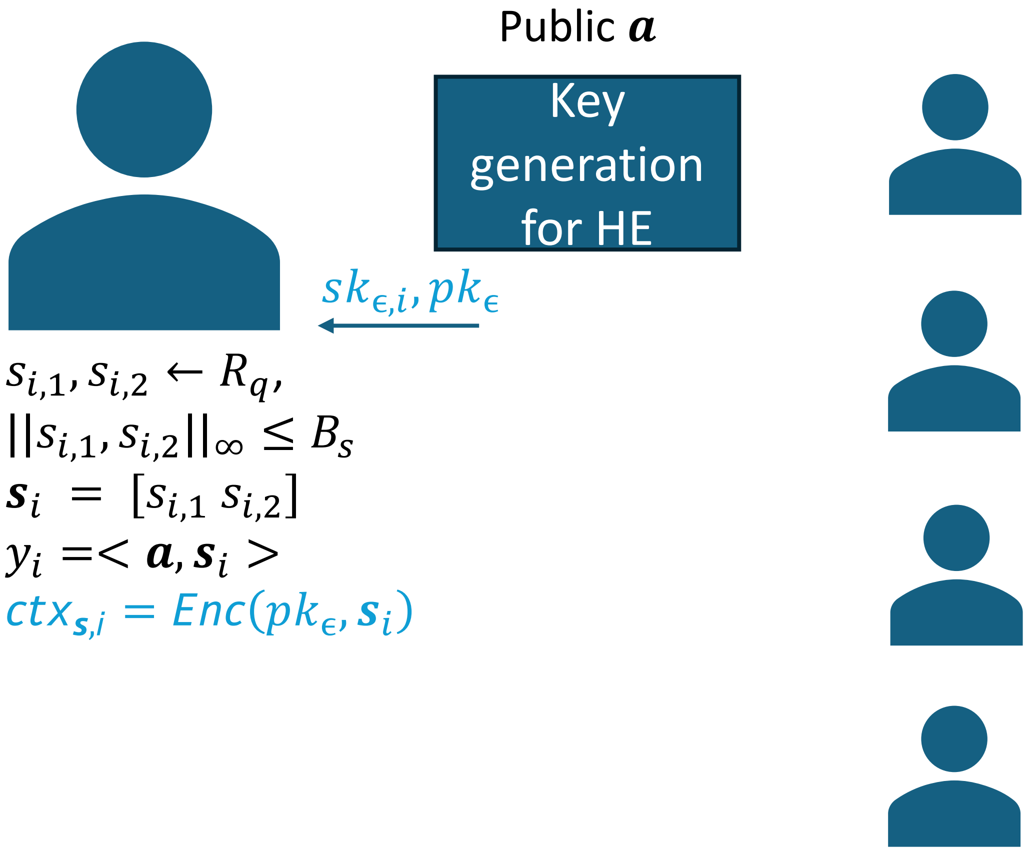
Building Signatures: Key Generation



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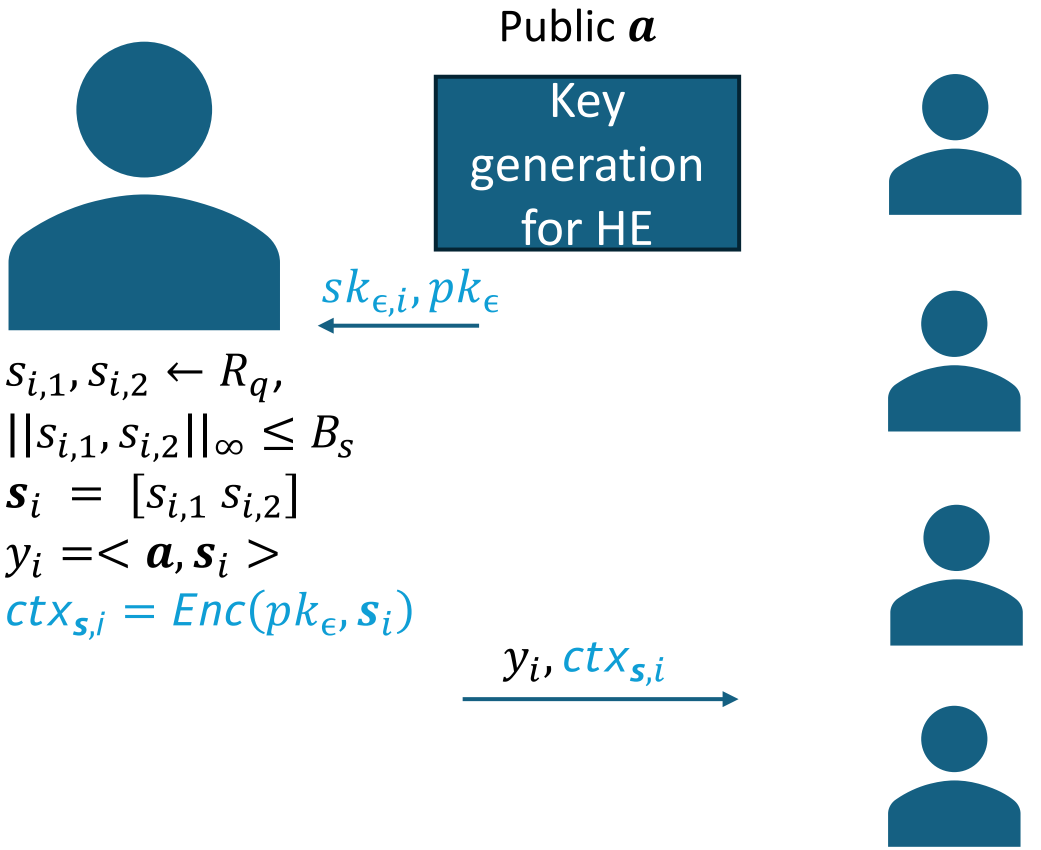

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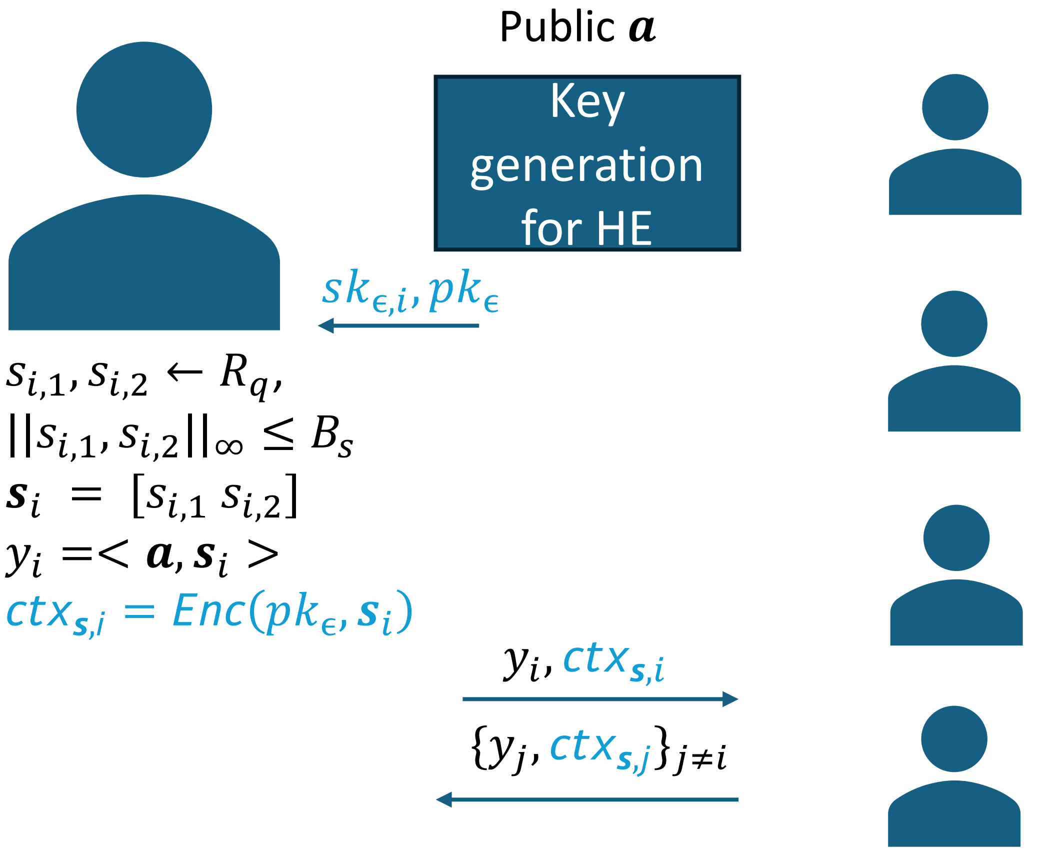

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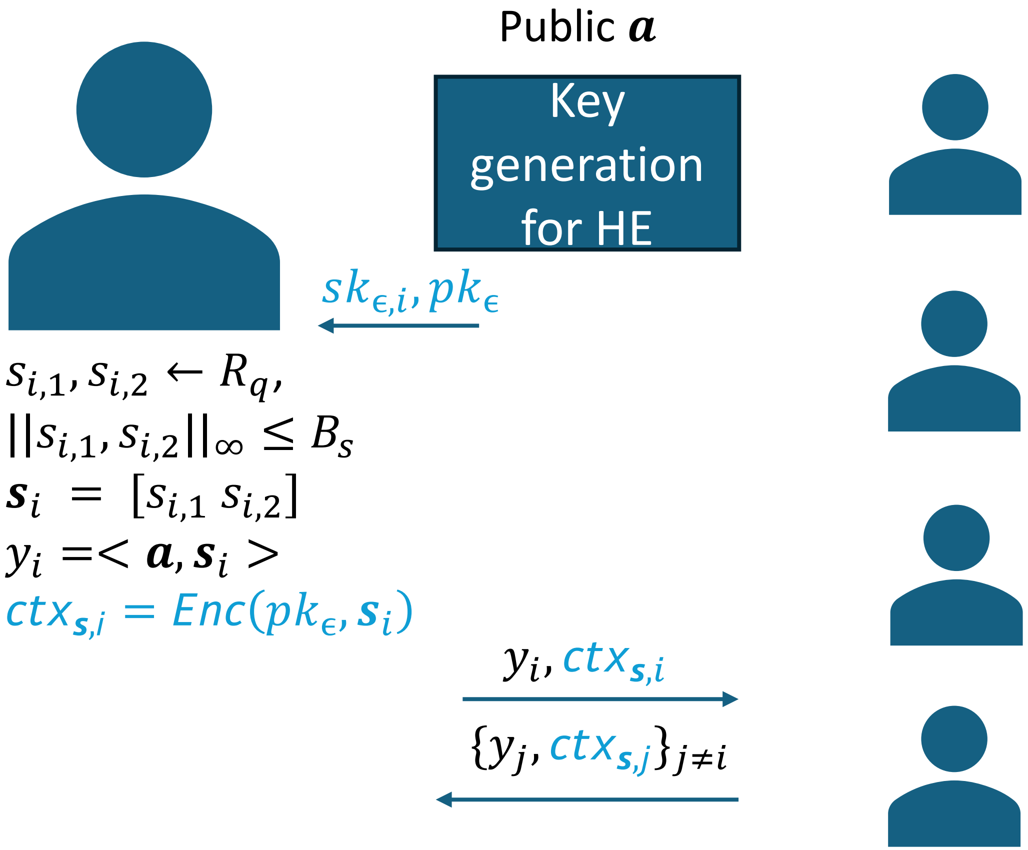

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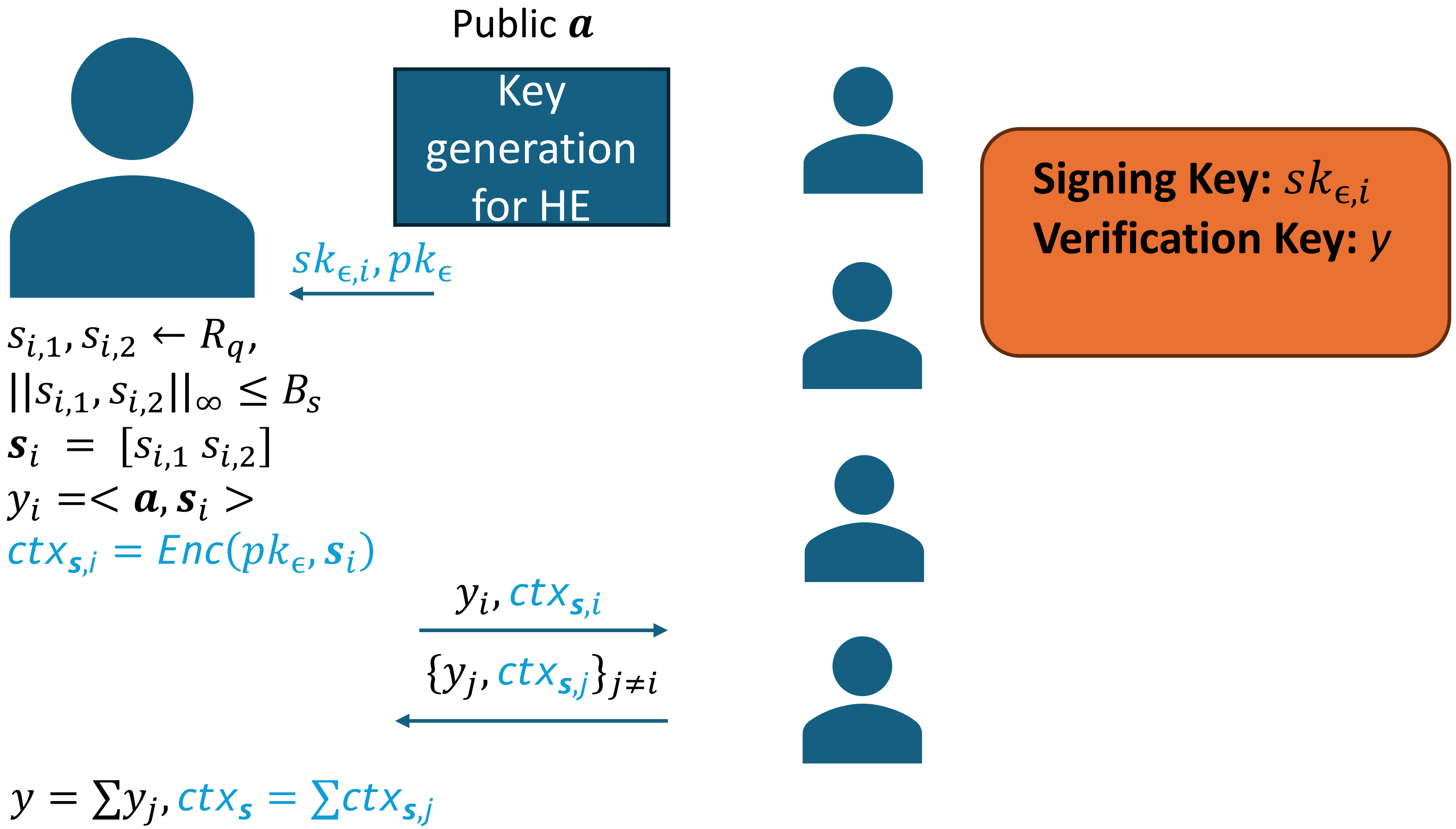
Building Signatures: Key Generation



Building Signatures: Key Generation

Public $\boldsymbol{a}$

Key generation for HE

$sk_{\epsilon,i}, pk_{\epsilon}$

$s_{i,1}, s_{i,2} \leftarrow R_q,$
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$\boldsymbol{s}_i = [s_{i,1}\ s_{i,2}]$
$y_i = < \boldsymbol{a}, \boldsymbol{s}_i >$
$ctx_{\boldsymbol{s},i} = Enc(pk_{\epsilon}, \boldsymbol{s}_i)$

$y_i, ctx_{\boldsymbol{s},i}$

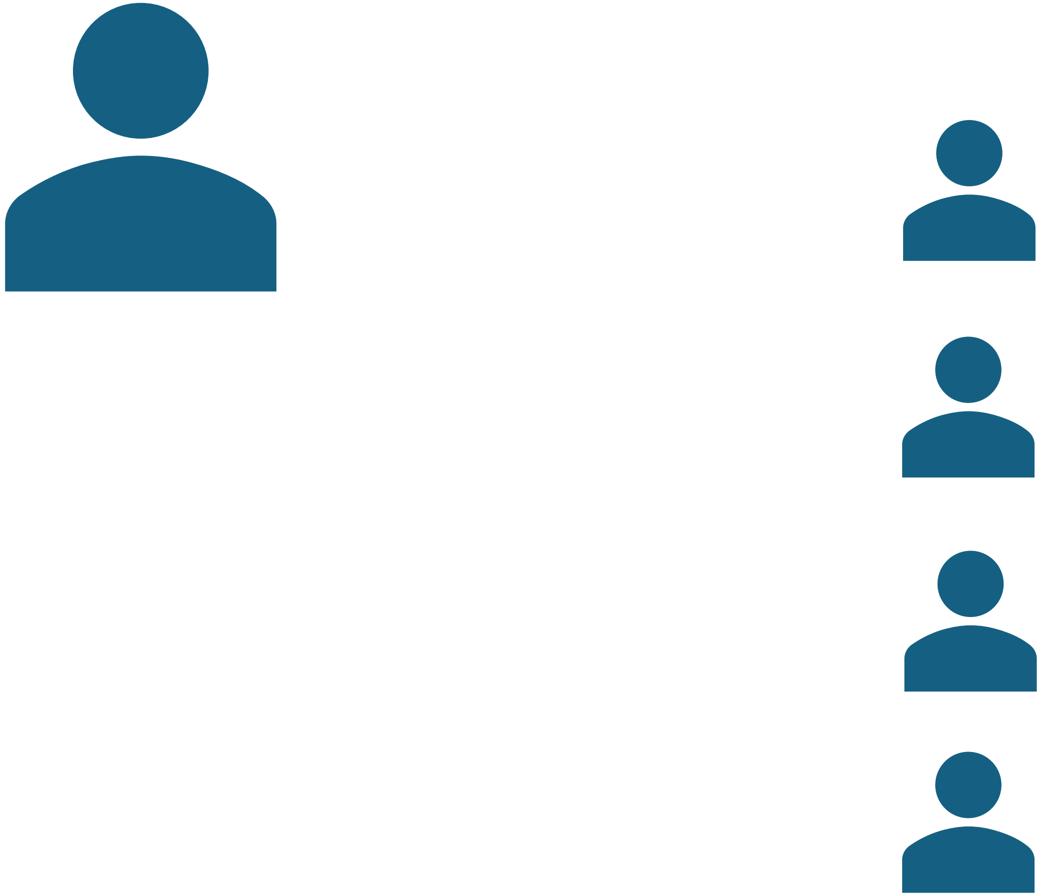
$\{y_j, ctx_{\boldsymbol{s},j}\}_{j \neq i}$

$y = \sum y_j, ctx_{\boldsymbol{s}} = \sum ctx_{\boldsymbol{s},j}$

Signing Key: $sk_{\epsilon,i}$
Verification Key: y

Commitments and ZKPs for active security

Building Signatures: Signing



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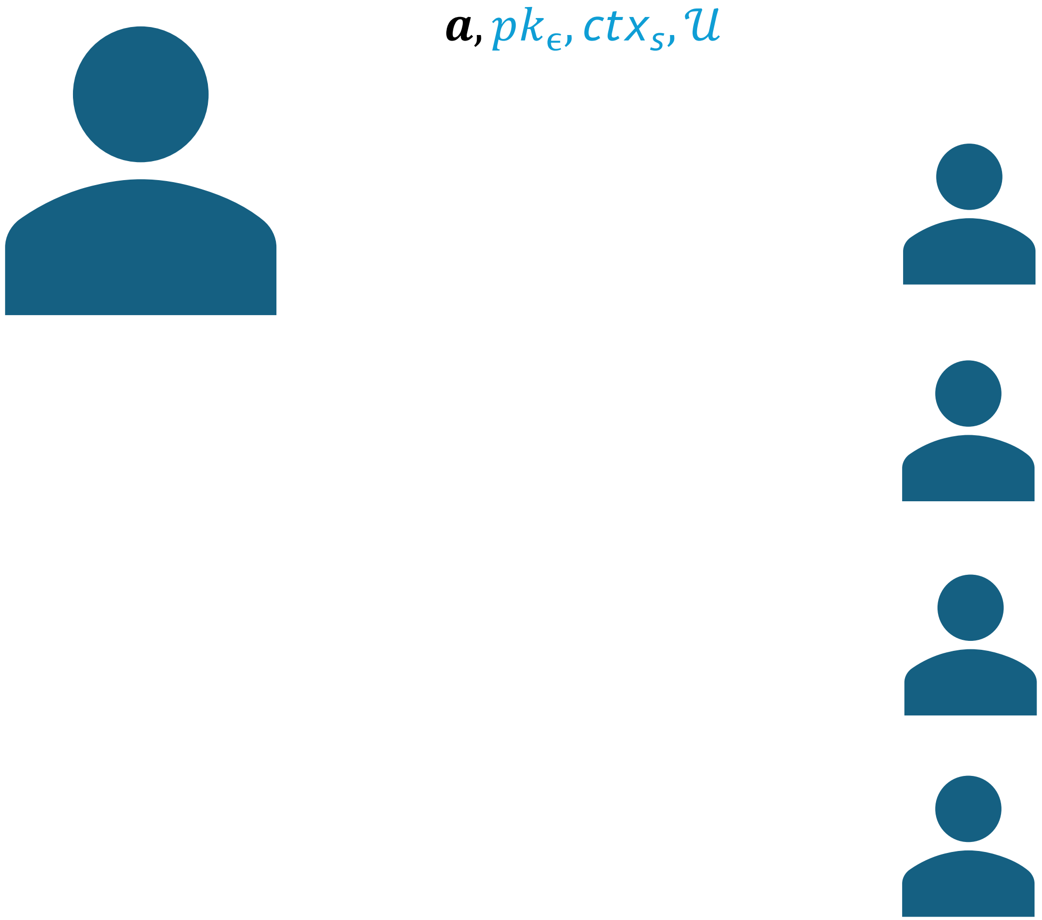

Building Signatures: Signing

$\boldsymbol{a}, pk_{\epsilon}, ctx_s, \mathcal{U}$

$r_{i,1}, r_{i,2} \leftarrow R_q, ||r_{i,1}, r_{i,2}||_{\infty} \leq B_s$

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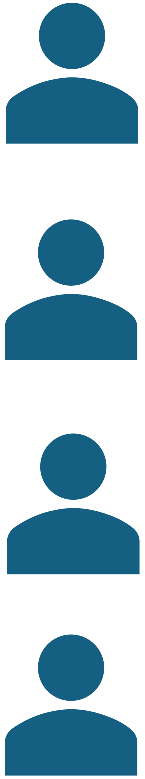
$ck = H(y, \mu), \rho_i \leftarrow \{0,1\}^*$

$com_i = Com_{ck}(w_i; \rho_i)$



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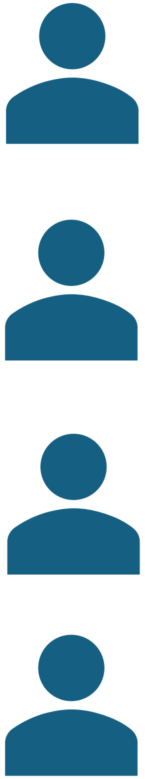

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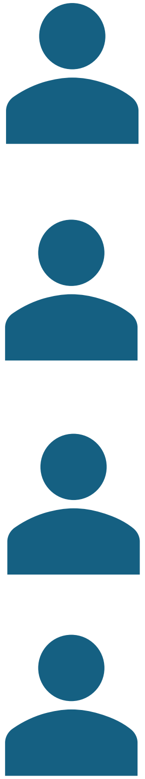

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$com_i, ctx_{\boldsymbol{r},i}$ →

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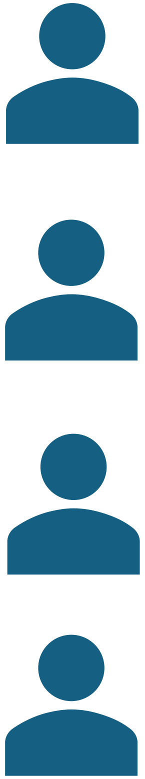

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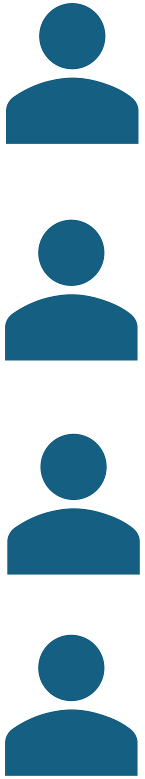

$c = H(\sum com_i\ , \mu)$

$ctx_{\boldsymbol{z}} = c\ ctx_{\boldsymbol{s}} + \sum ctx_{\boldsymbol{r},j}$

Building Signatures: Signing

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$\{com_j, ctx_{\boldsymbol{s},j}\}_{j \neq i}$ ←

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$ds_i = TDec(ctx_{\boldsymbol{z}}, sk_{\epsilon,i}, \mathcal{U})$

Building Signatures: Signing

$\boldsymbol{a}, pk_\epsilon, ctx_s, \mathcal{U}$

$r_{i,1}, r_{i,2} \leftarrow R_q, ||r_{i,1}, r_{i,2}||_\infty \le B_s$

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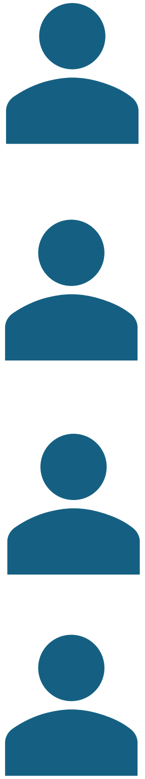
w_i, ρ_i, ds_i →



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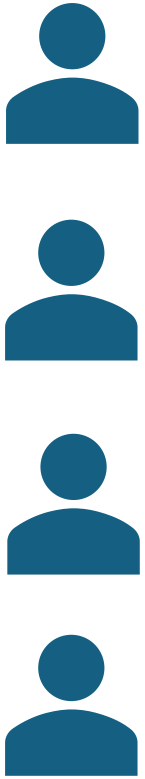
w_i, ρ_i, ds_i →

$\{w_i, \rho_i, ds_i\}_{j \neq i}$ ←



Building Signatures: Signing

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$ck = H(y, \mu), \rho_i \leftarrow \{0,1\}^*$

$com_i = Com_{ck}(w_i; \rho_i)$

$ctx_{\boldsymbol{r},i} = Enc(pk_\epsilon, \boldsymbol{r}_i)$

$com_i, ctx_{\boldsymbol{r},i} \rightarrow$

$\leftarrow \{com_j, ctx_{\boldsymbol{s},j}\}_{j \neq i}$

$c = H(\sum com_i, \mu)$

$ctx_{\boldsymbol{z}} = c\ ctx_{\boldsymbol{s}} + \sum ctx_{\boldsymbol{r},j}$

$ds_i = TDec(ctx_{\boldsymbol{z}}, sk_{\epsilon,i}, \mathcal{U})$

$w_i, \rho_i, ds_i \rightarrow$

$\leftarrow \{w_i, \rho_i, ds_i\}_{j \neq i}$

$\boldsymbol{z} = Comb(ctx_{\boldsymbol{z}}, \{ds_j\}, \mathcal{U}), \rho = \sum \rho_i$

Building Signatures: Signing

$\boldsymbol{a}, pk_{\epsilon}, ctx_{s}, \mathcal{U}$

$r_{i,1}, r_{i,2} \leftarrow R_q, ||r_{i,1}, r_{i,2}||_{\infty} \leq B_s$

$\boldsymbol{r}_i = [r_{i,1}\ r_{i,2}], w_i = < \boldsymbol{a}, \boldsymbol{r}_i >$

$ck = H(y, \mu), \rho_i \leftarrow \{0,1\}^*$

$com_i = Com_{ck}(w_i; \rho_i)$

$ctx_{\boldsymbol{r},i} = Enc(pk_{\epsilon}, \boldsymbol{r}_i)$

$com_i, ctx_{\boldsymbol{r},i}$

$\{com_j, ctx_{\boldsymbol{s},j}\}_{j \neq i}$

$c = H(\sum com_i, \mu)$

$ctx_{\boldsymbol{z}} = c\ ctx_{\boldsymbol{s}} + \sum ctx_{\boldsymbol{r},j}$

$ds_i = TDec(ctx_{\boldsymbol{z}}, sk_{\epsilon,i}, \mathcal{U})$

w_i, ρ_i, ds_i

$\{w_i, \rho_i, ds_i\}_{j \neq i}$

$\boldsymbol{z} = Comb(ctx_{\boldsymbol{z}}, \{ds_j\}, \mathcal{U}), \rho = \sum \rho_i$

Signature: $(c, \boldsymbol{z}, \rho)$

Building Signatures: Signing

$\boldsymbol{a}, pk_\epsilon, ctx_s, \mathcal{U}$

$r_{i,1}, r_{i,2} \leftarrow R_q, ||r_{i,1}, r_{i,2}||_\infty \leq B_s$

$\boldsymbol{r}_i = [r_{i,1}\ r_{i,2}], w_i = < \boldsymbol{a}, \boldsymbol{r}_i >$

$ck = H(y, \mu), \rho_i \leftarrow \{0,1\}^*$

$com_i = Com_{ck}(w_i; \rho_i)$

$ctx_{\boldsymbol{r},i} = Enc(pk_\epsilon, \boldsymbol{r}_i)$

$com_i, ctx_{\boldsymbol{r},i}$ →

← $\{com_j, ctx_{\boldsymbol{s},j}\}_{j \neq i}$

$c = H(\sum com_i\ , \mu)$

$ctx_{\boldsymbol{z}} = c\ ctx_{\boldsymbol{s}} + \sum ctx_{\boldsymbol{r},j}$

$ds_i = TDec(ctx_{\boldsymbol{z}}, sk_{\epsilon,i}, \mathcal{U})$

w_i, ρ_i, ds_i →

← $\{w_i, \rho_i, ds_i\}_{j \neq i}$

$\boldsymbol{z} = Comb(ctx_{\boldsymbol{z}}, \{ds_j\}, \mathcal{U}), \rho = \sum \rho_i$

Signature: $(c, \boldsymbol{z}, \rho)$

Commitments and ZKPs for active security

- Security?

- Security?
  - Threshold unforgeability from underlying unforgeability + HE indistinguishability + RLWE + commitment/ZKP security(active only)

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- Efficiency?

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- Efficiency?

Scheme, # of Signatures	Public key (KB)	Signature (KB)	# of Rounds	Distributed Key Generation	Identifiable Abort
This work, β=1	2.6	8.5	2	✓	✓
This work, β=365	3.1	10.4	2	✓	✓
This work, $β=2^{64}$	13.6	46.6	2	✓	✓
TRaccoon [dPKM+24]*, $β=2^{60}$	3.9	12.7	3	✗**	✗

* = Public after our submission
**= Can use ours

Conclusion

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  - Supports distributed key generation + identifiable abort
  - Simple linear HE with distributed key generation as a building block
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- Future Work
  - Protocol optimizations
  - Adaptive security
  - Same framework, different problems

Thank You!

Full Version:
https://ia.cr/2023/1318

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