



MATHEMATICAL
INSTITUTE

2nd Oxford
**Post-Quantum
Cryptography**
Summit 2023

 PQ SHIELD



National Cyber
Security Centre

Andrew Wiles Building

Mathematical Institute

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VOX and PROV

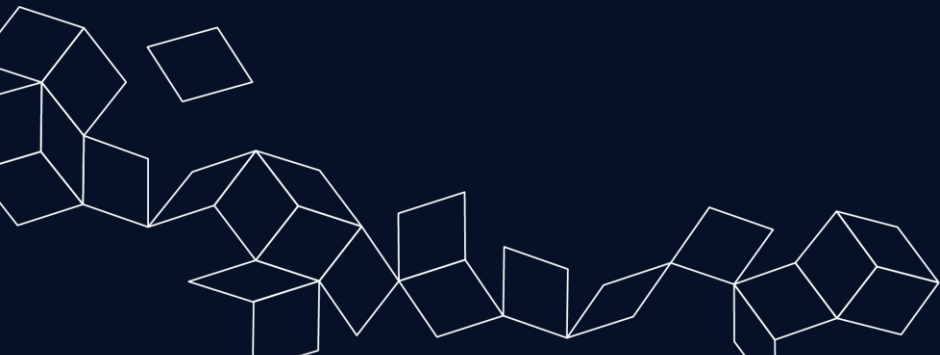
Benoît Cogliati, Jean-Charles Faugère, Pierre-Alain Fouque, Louis Goubin, Robin Larrieu, Gilles Macario-Rat, Brice Minaud, Jacques Patarin

Orange, Thales, Versailles St-Quentin University, Rennes University, CryptoNext



VOX

- What it is
- How it is made
- How it works
- How nice it is



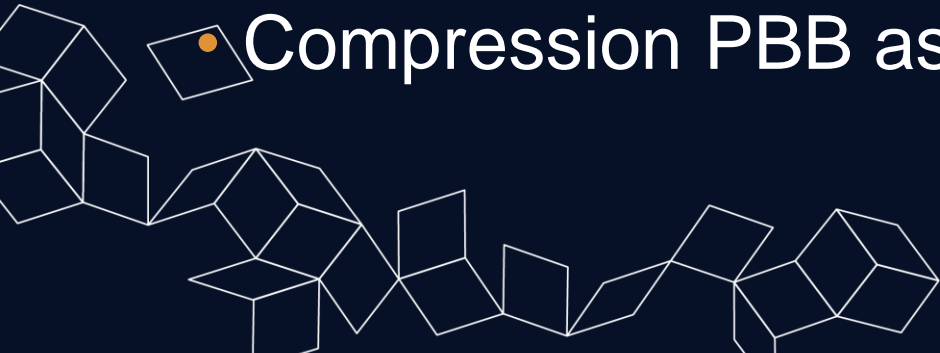
Features of VOX

- VOX : like UOV, but better !
- UOV ($n=o+v, m=o, q$) based
- Hat Plus : t secret equations are set as complete random
- o and v have a common divider $l \Rightarrow$ enables QR transformation
- Public and Secret compression (for free)



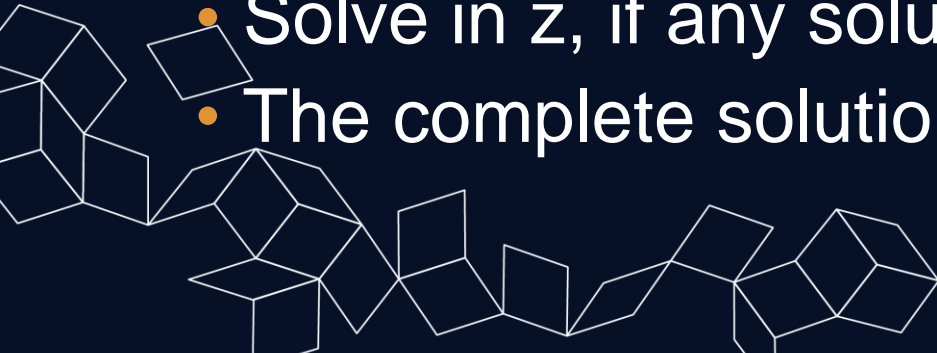
Key Generation

- Generate UOV ($n=o/l+v/l, m=o, q^l$) secret key
 - Note: $\text{Min}(l)=1$ corresponds to plain UOV ; $\text{Max}(l) = \text{GCD}(o,v)$
- Complete the last t equations with random coefficients
 - Note $\text{Min}(t)=0$ does nothing ; $\text{Max}(t)=m$ is a complete random key
- Add S in $\text{Pub} = S \circ \text{Sec} \circ T$ (S in F_q , T in F_q^l)
- You get: $Vox(n=o+v, m=o, q, t, l)$
- Compression PBB as with UOV



Inversion of the secret map

- Fix vinegar variables with random values : B
- The first $o-t$ equations represent a $(o, o-t)$ - linear system
 - Its solutions can (with great probability) be described as an affine space of dimension t
 - $A = A_0 + z_1 A_1 + \dots + z_t A_t$ with free variables z_1, \dots, z_t
- Substitute in the last t equations, to get a (t, t) – quadratic system
- Solve in z , if any solution then substitute in A
- The complete solution is then $A \parallel B$



Signature

- Hash and Sign
- Apply S^{-1}
- Invert the secret map, until a solution is found (≈ 1.5 times in average)
 - The $(o, o-t)$ linear system has $(1 - 1/q^{t+1})$ probability to be regular
 - The (t, t) quadratic system has $\approx (1 - t/q)$ to be regular with at least one solution

- Apply T^{-1}



Attacks

- Direct attack
 - Choose parameter against MQ estimator
- Distinguishing attacks (UOV)
 - Choose $q^{3t} \geq 2^\lambda$
- Structural attacks : Rectangular MinRank attack
(pqc forum : Comment from Hiroki Furue)
 - Choose o, v, t, l such that $\text{Min}(o, v/l + t) \geq \text{Min}(o, v/l + o/l)$ (i.e. $t \geq o/l$)



Performance

- Example Level I
- $q = 251$
- $o/l = 6$
- $v/l = 7$
- $t = 6$
- $l = 8$
- $|\text{SIG}| = 104 \text{ B}$
- $|\text{PK}| = 7,088 \text{ B}$
- $|\text{SK}| = 27,952$
- KeyGen = 390 μs
- Sign = 270 μs
- Verify = 20 μs



Features of Prov

- UOV based
- δ equations are removed
- Salted
- BUFF tweak
- Public and Secret key compression (for free)
- Main objective : Security proof !



Inversion of the secret map

- With high probability, the $(o, o-\delta)$ linear system is regular, hence it has q^δ solutions.
- The signatures are (almost) uniformly distributed.



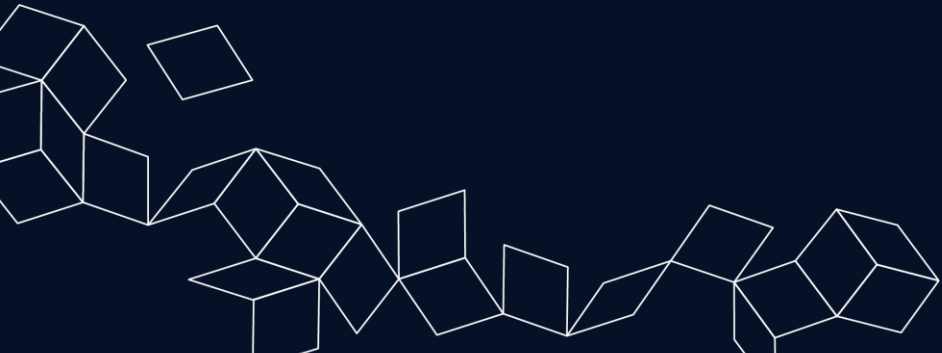
Signature

- The signature is a solution in x of
 - $\text{Pub}(x) = \text{Hash}(\text{Hpk} \parallel \text{message} \parallel \text{salt})$
- Hpk is a hash of the public key (BUFF tweak)
- Vinegar is drawn at random only once
- Salt is drawn at random until the derived linear system has a solution



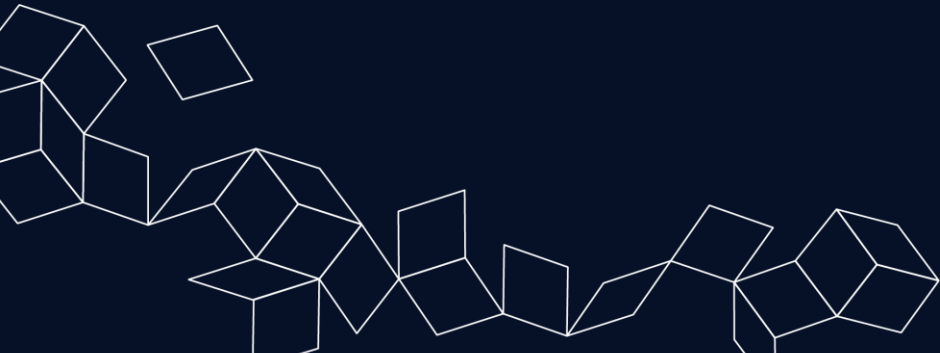
Security of Signature

- The property of EUF-CMA of PROV can be proven in both classical and Quantum random Oracle Model, provided the problem on inverting PROV is hard.



Performance

- Example Level I
- $q = 256$
- $n = o + v = 136$
- $m = o - \delta = 46$
- $\delta = 8$
- $|\text{SIG}| = 160 \text{ B}$
- $|\text{PK}| = 68,326 \text{ B}$
- $|\text{SK}| = 203,752 \text{ B}$



Sites

- <http://vox-sign.com>
- <http://prov-sign.github.io>

- Thank You !

- Dankon !



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