Rank Metric in the Head

RYDE

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn, T. Feneuil, P. Gaborit, A. Joux, M. Rivain, J.-P. Tillich, A. Vincotte

MIRA

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2nd Oxford Post-Quantum Cryptography Summit

September 4, 2023, University of Oxford (UK)

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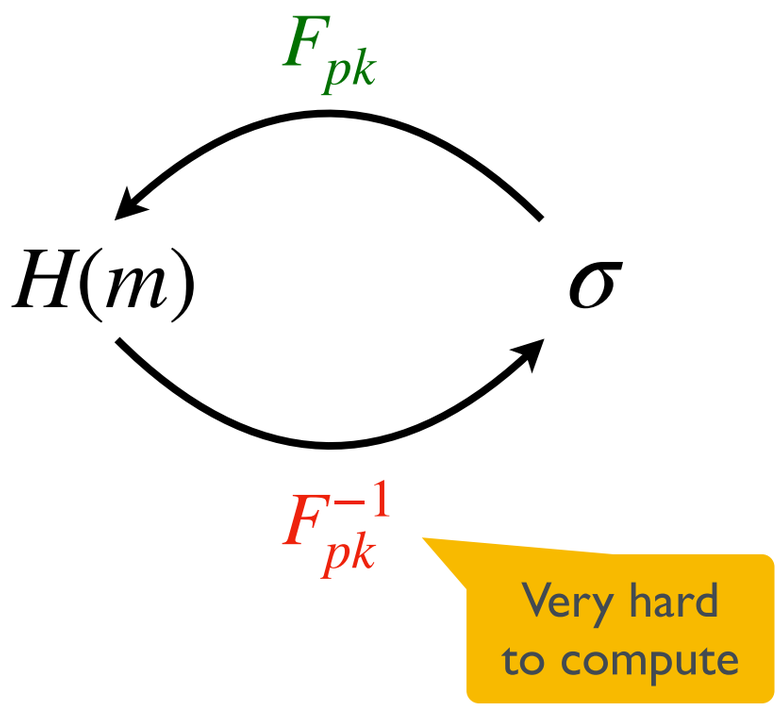
Introduction

How to build signature schemes?

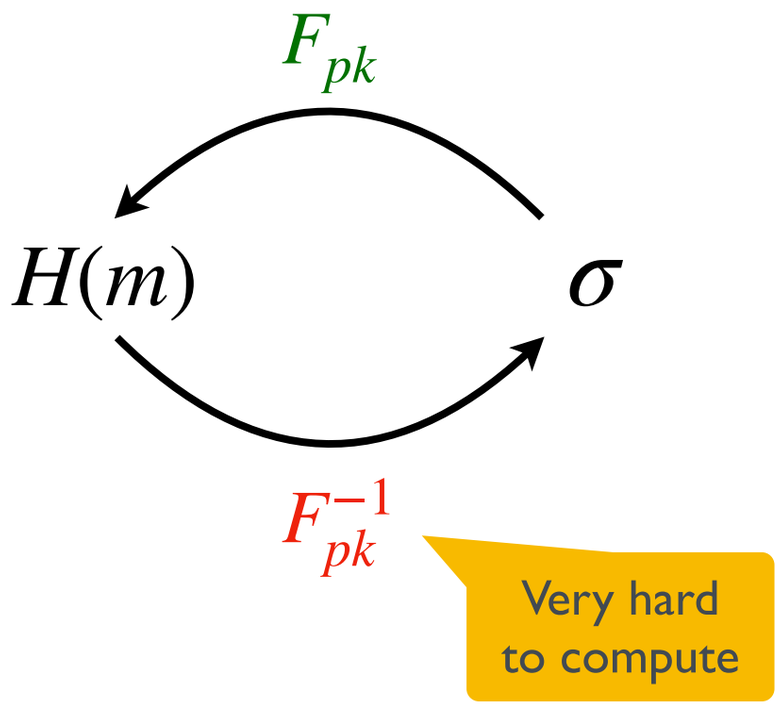
Hash & Sign



- Short signatures
- “Trapdoor” in the public key

How to build signature schemes?

Hash & Sign



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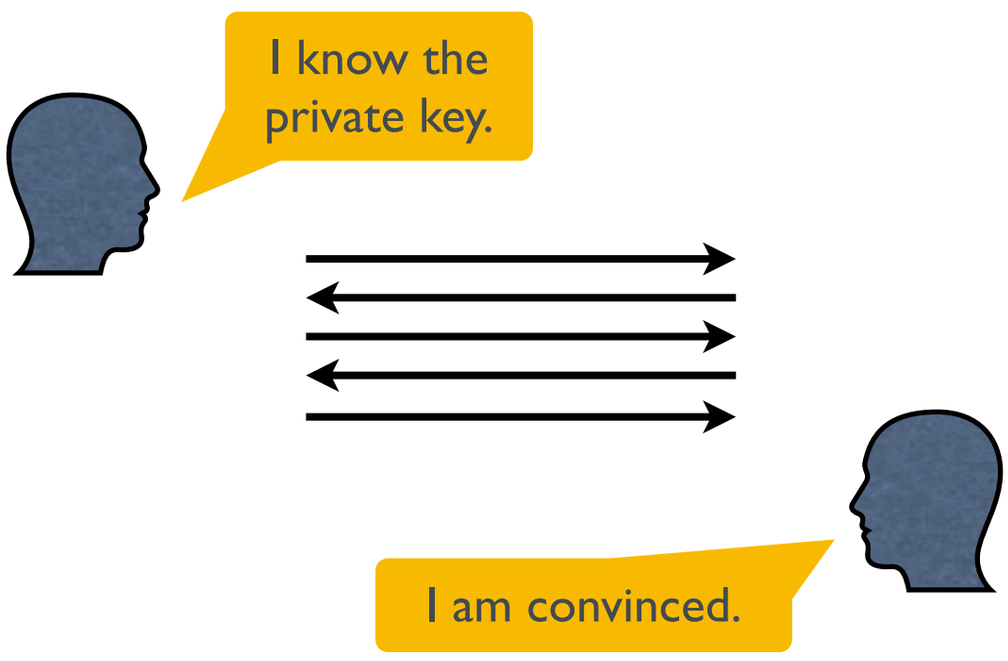
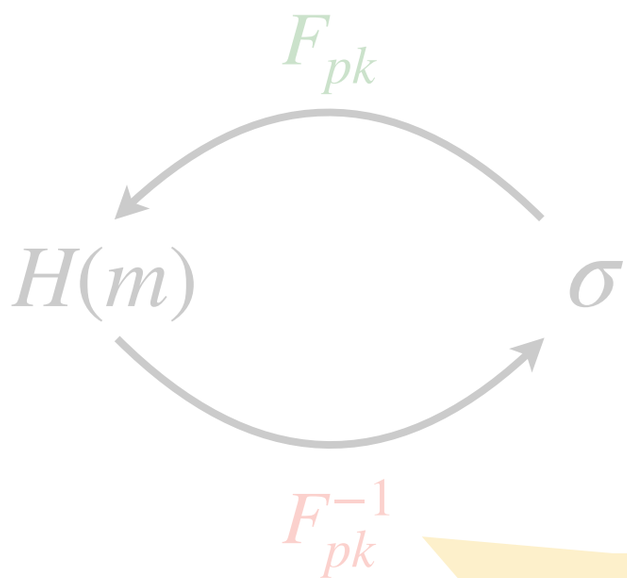
From a zero-knowledge proof



- Large(r) signatures
- Short public key

How to build signature schemes?

Hash & Sign



Very hard to compute

- Short signatures
- "Trapdoor" in the public key

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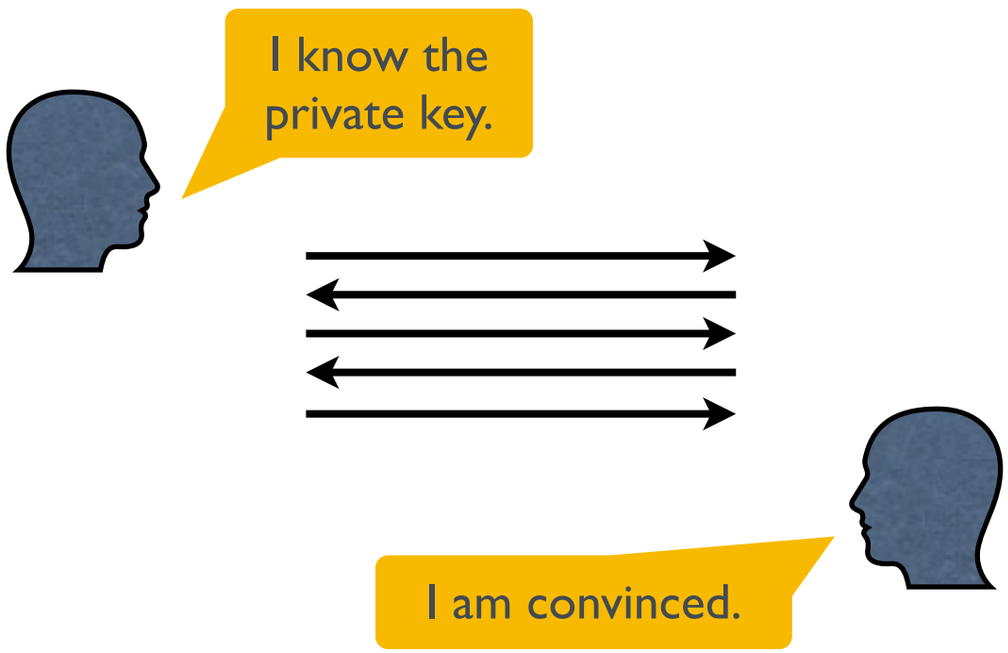
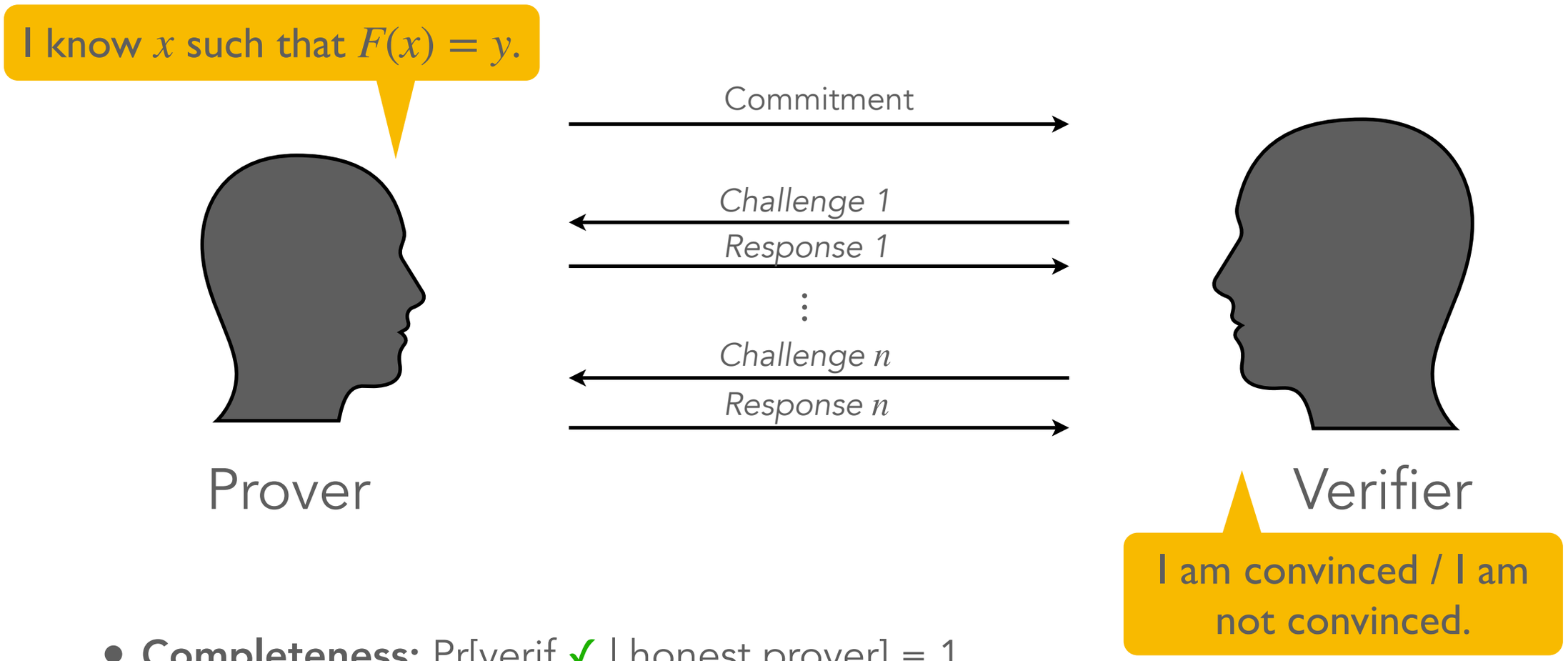


- Large(r) signatures
- Short public key

Proof of knowledge



- **Completeness:** Pr[verif ✓ | honest prover] = 1
- **Soundness:** Pr[verif ✓ | malicious prover] $\leq \varepsilon$ (*e.g.* 2^{-128})
- **Zero-knowledge:** verifier learns nothing on x

MPC in the Head

- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be applied to any cryptographic problem

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Figure: Number of citations to [IKOS07] by year

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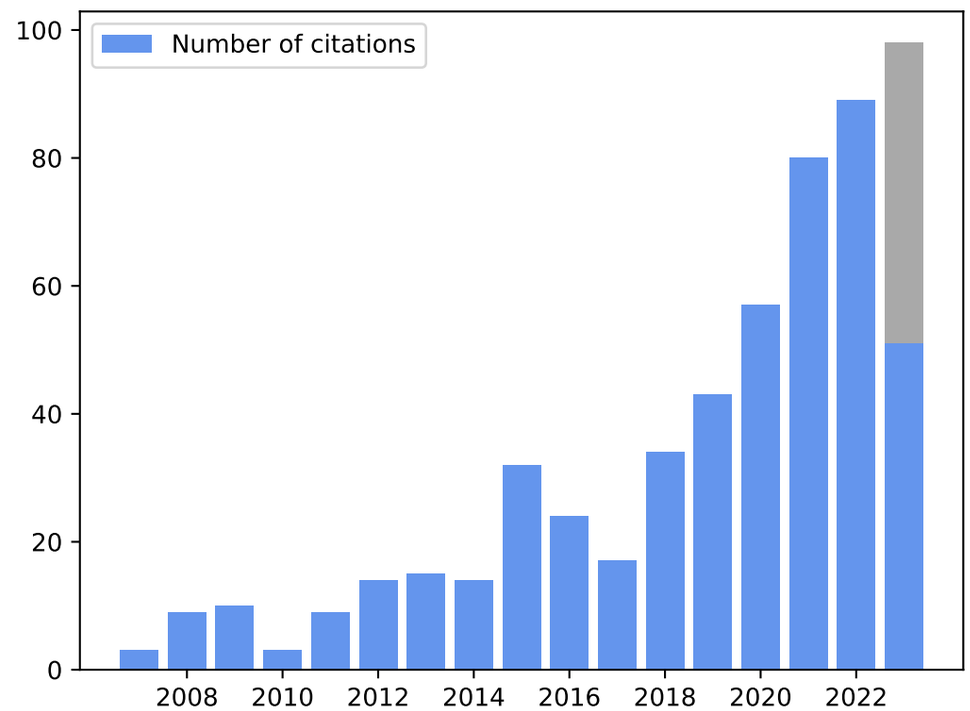

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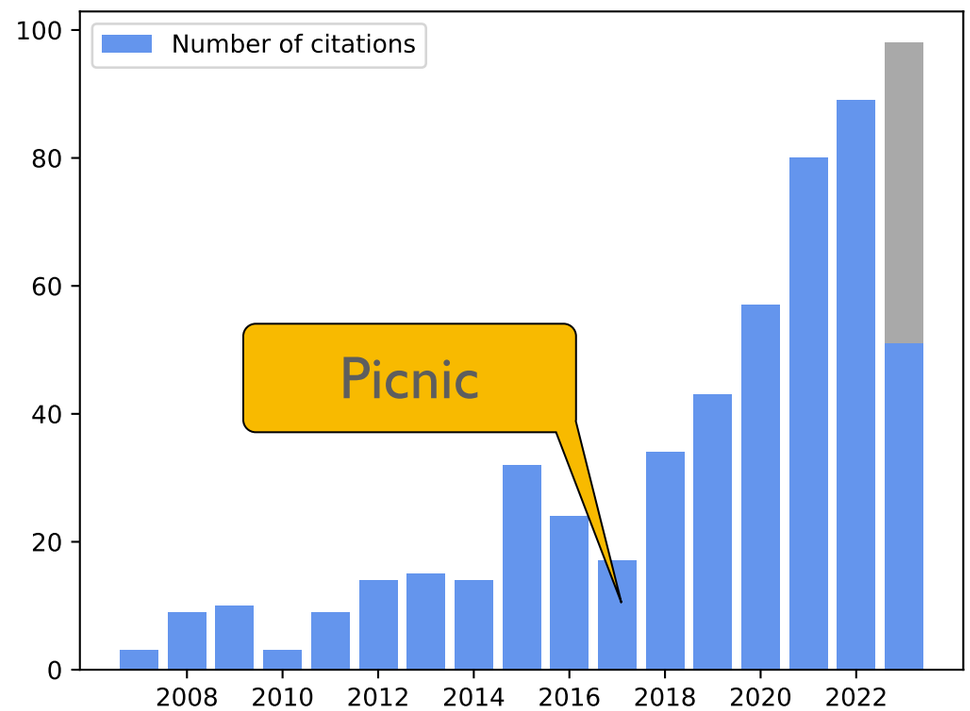

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- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic**: can be applied to any cryptographic problem
- Convenient to build (candidate) **post-quantum signature** schemes
- **Picnic**: submission to NIST (2017)
- First round of recent NIST call: 8 MPCitH schemes / 40 submissions

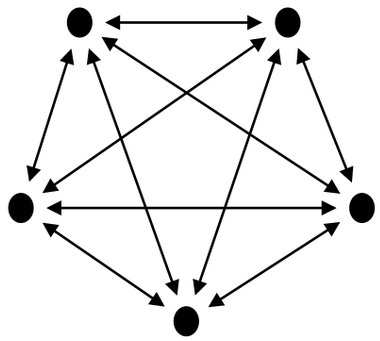
AIMer	*MQOM*
Biscuit	*PERK*
MIRA	*RYDE*
MiRitH	*SDitH*

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Multiparty computation (MPC)

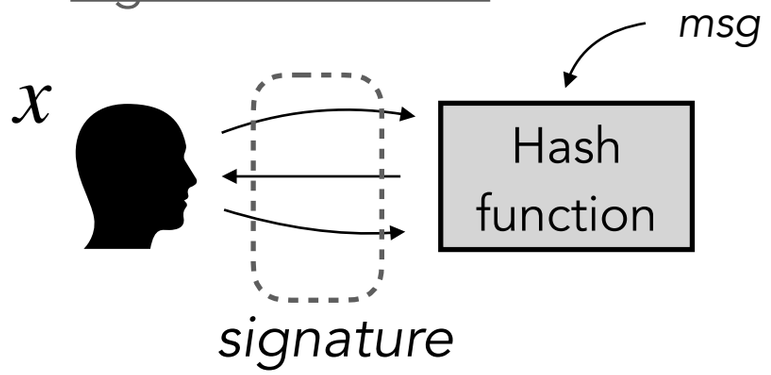


Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Signature scheme



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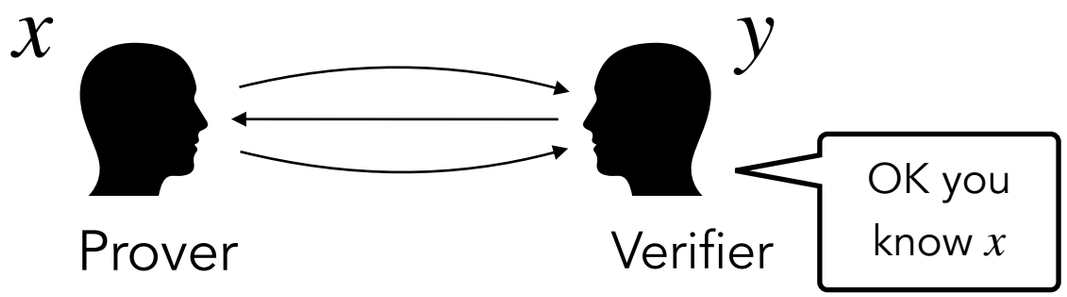

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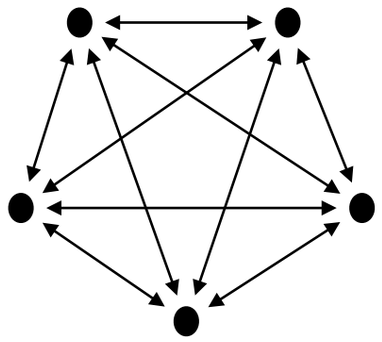

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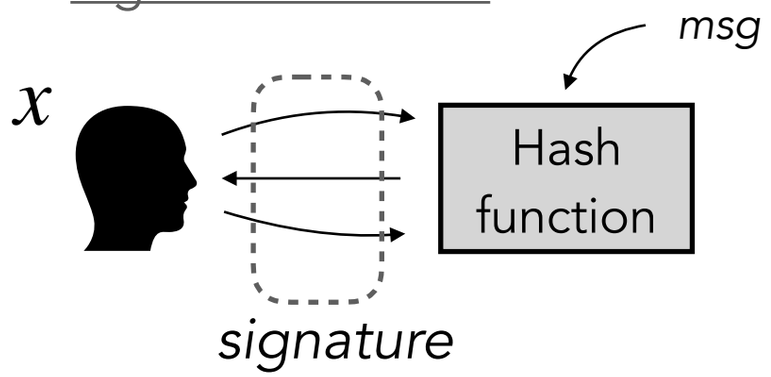

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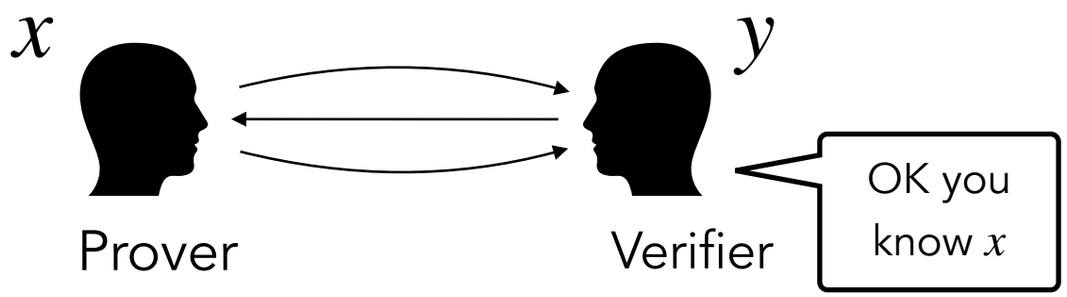

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$ s.t. $x = [\![x]\!]_1 + \ldots + [\![x]\!]_N$

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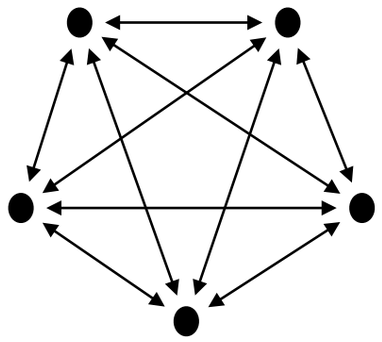

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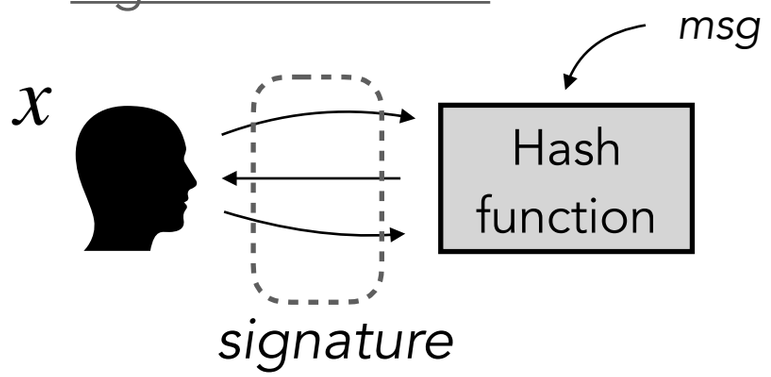

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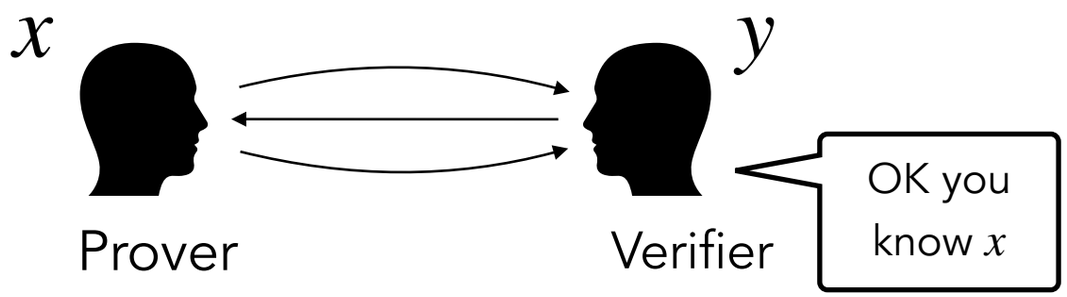

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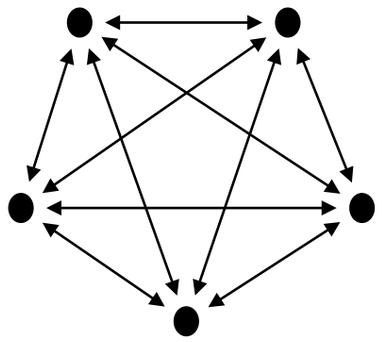

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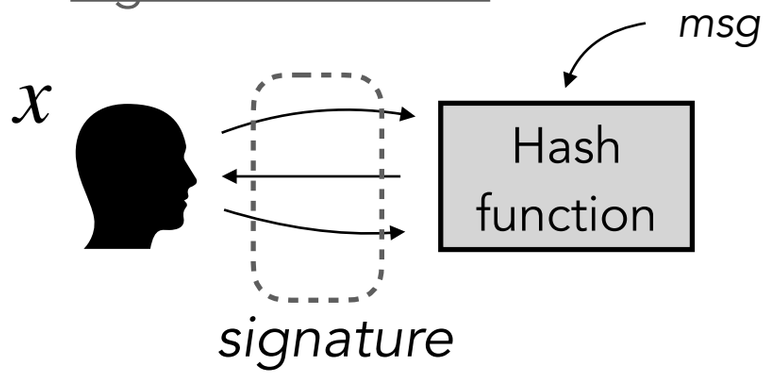

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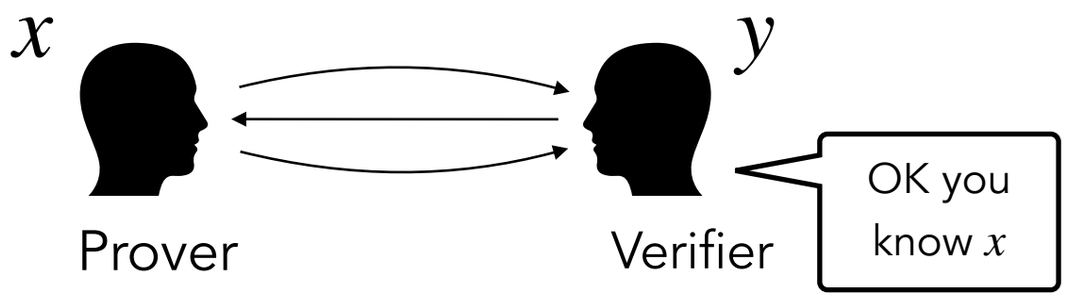

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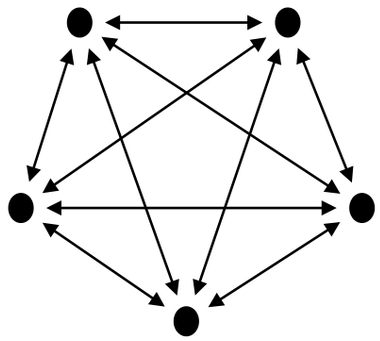

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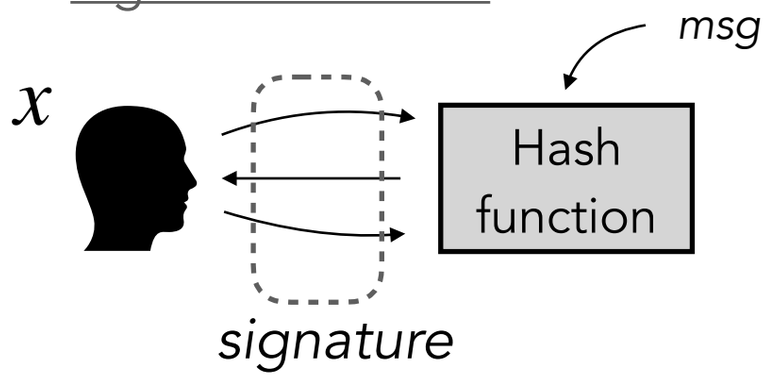

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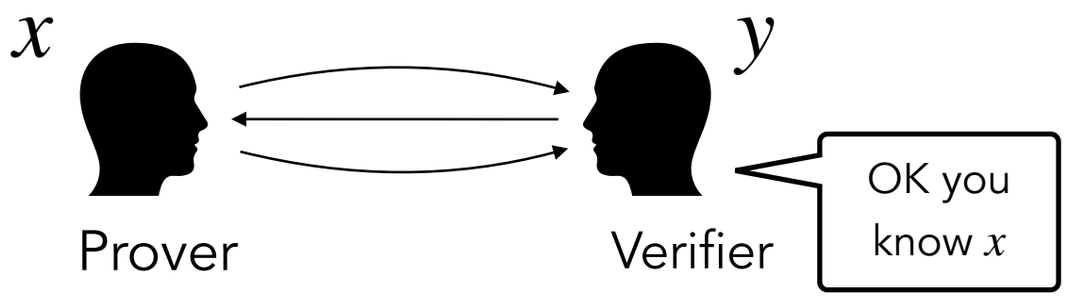

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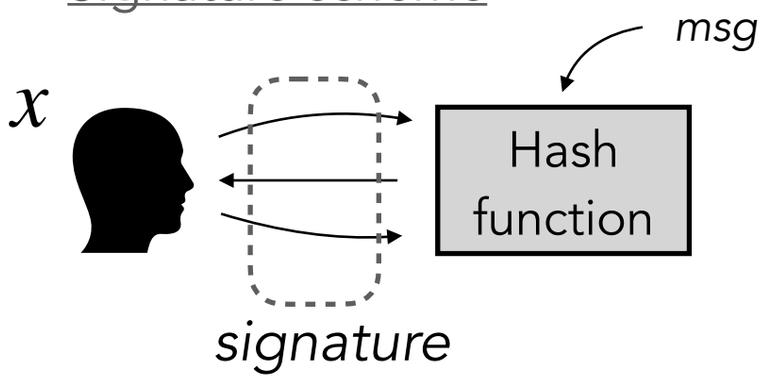

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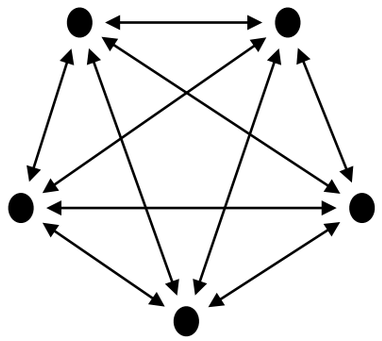

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MPC-in-the-Head transform

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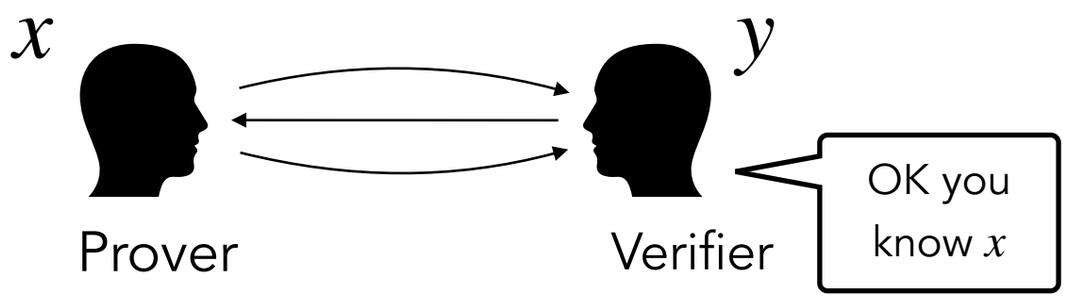
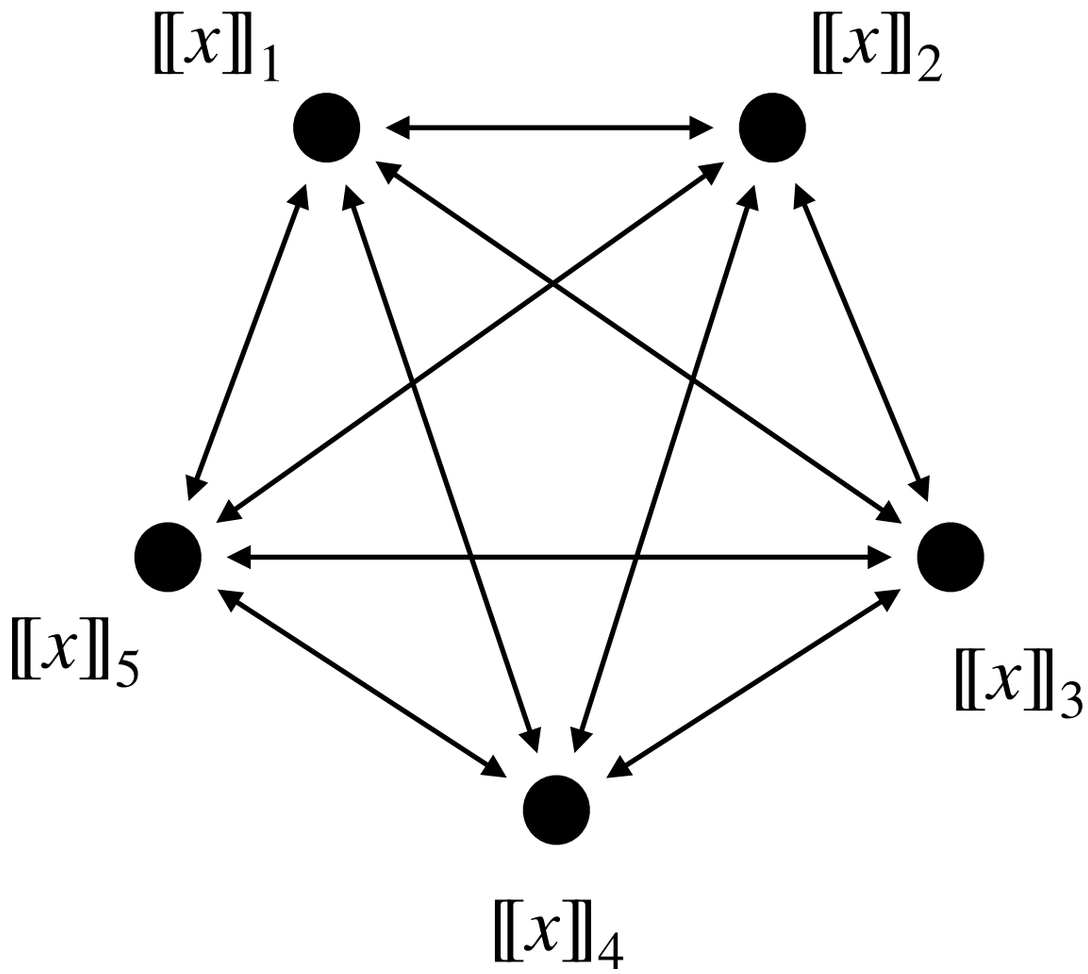


MPCitH: general principle

MPC model



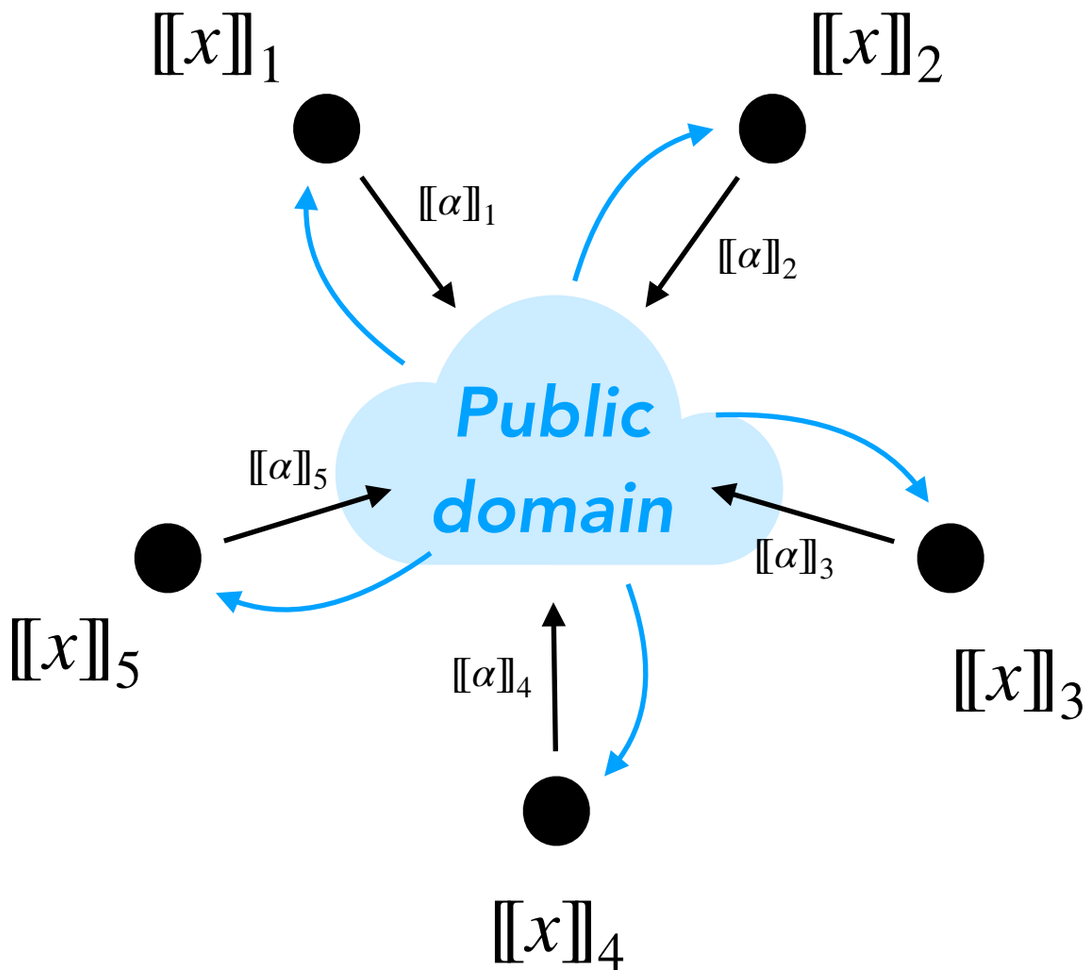
$$x = [\![x]\!]_1 + [\![x]\!]_2 + \ldots + [\![x]\!]_N$$

- **Jointly compute**

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol

MPC model



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- $(N-1)$ **private:** the views of any $N-1$ parties provide no information on x
- **Semi-honest model:** assuming that the parties follow the steps of the protocol
- ***Broadcast model***
  - *Parties locally compute on their shares* $[\![x]\!] \mapsto [\![\alpha]\!]$
  - *Parties broadcast* $[\![\alpha]\!]$ *and recompute* α
  - *Parties start again (now knowing* α*)*

MPCitH transform

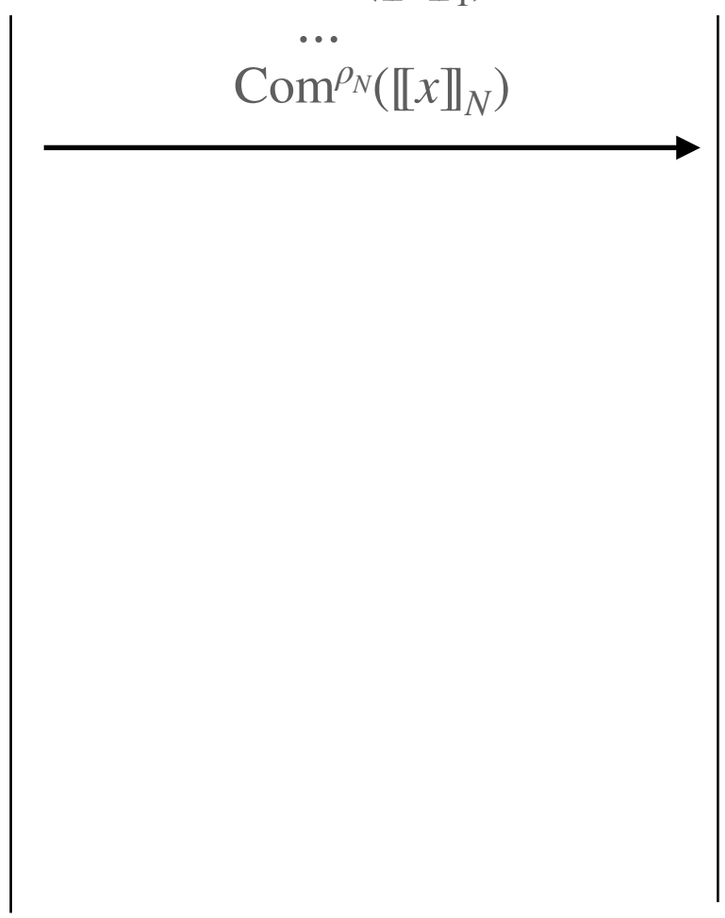
Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$



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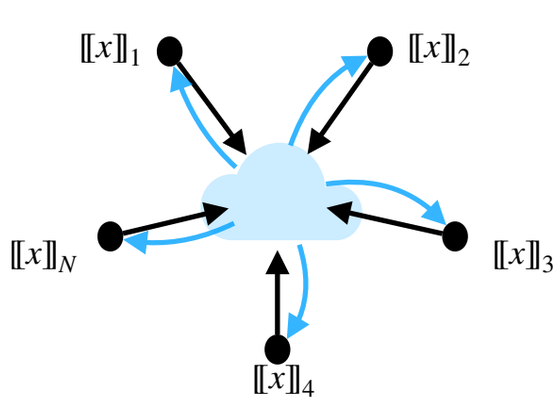
② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
…
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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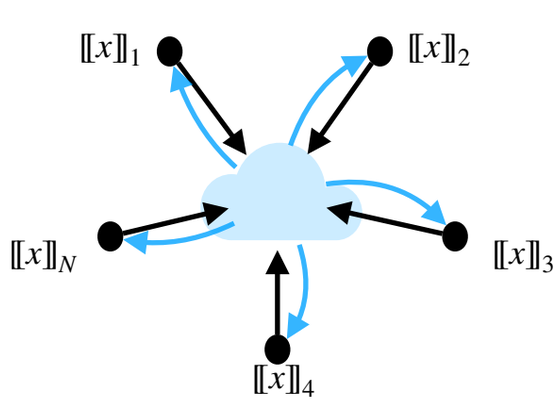

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i^*

③ Choose a random party

$i^* \leftarrow^{\$} \{1, \ldots, N\}$

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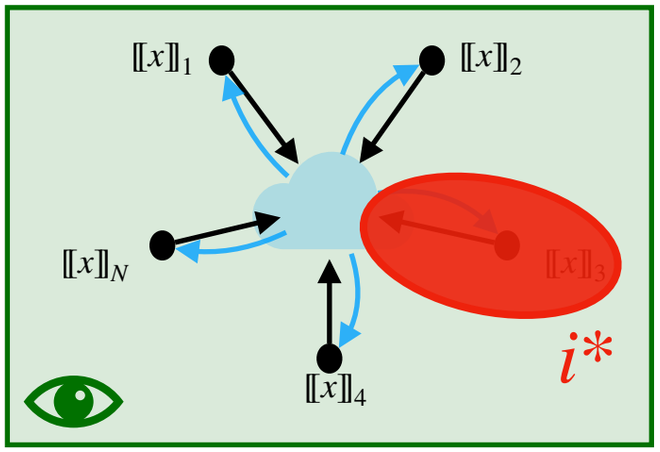

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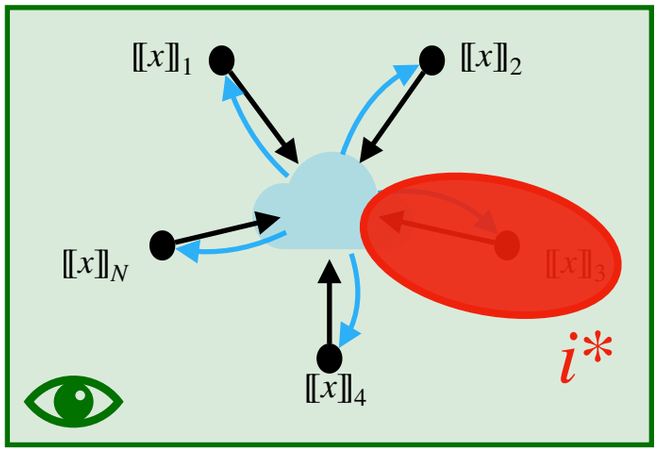

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⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

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We have $F(x) \neq y$ where

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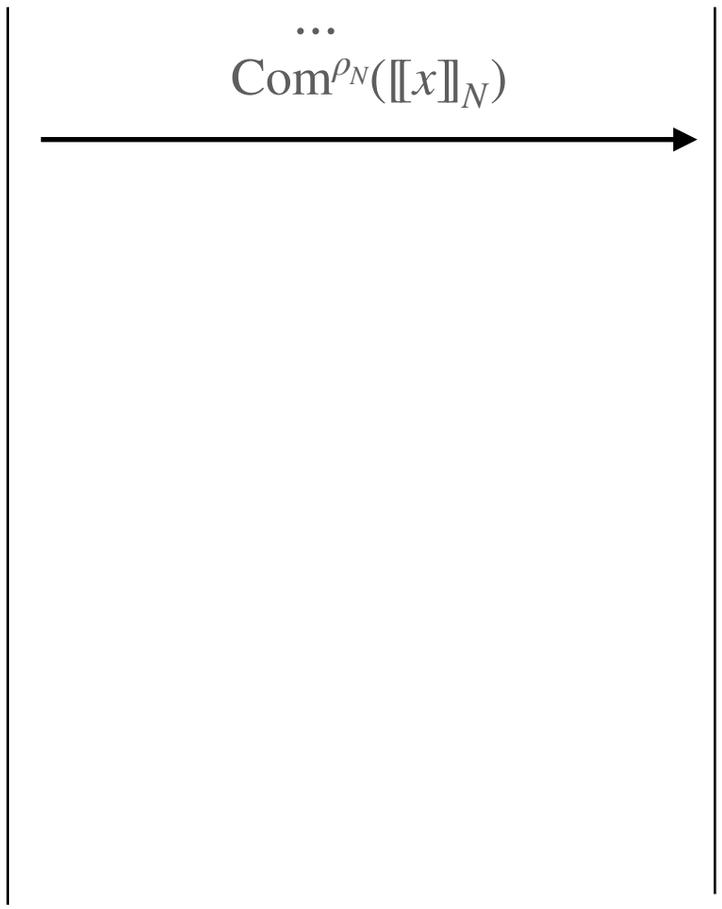

Malicious Prover Verifier

MPCitH transform

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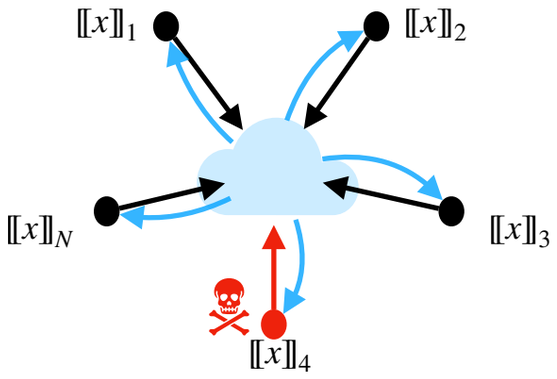

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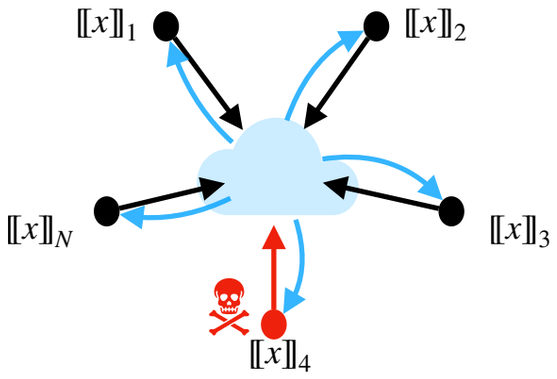

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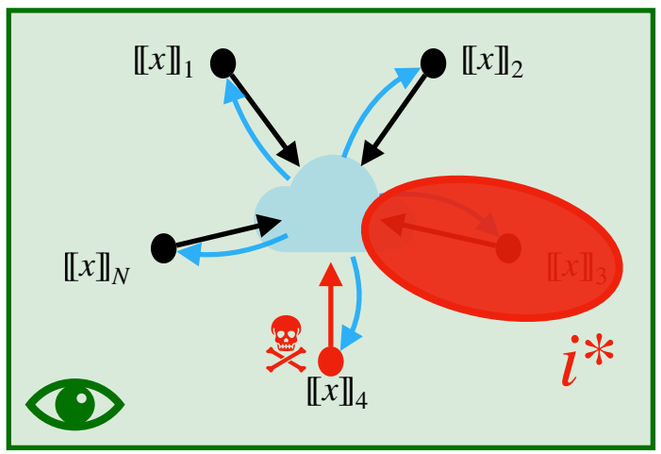


④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
$\cdots$
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

③ Choose a random party
$i^* \leftarrow^{\$} \{1,\ldots,N\}$

i^*

←

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

Malicious Prover

Verifier

MPCitH transform

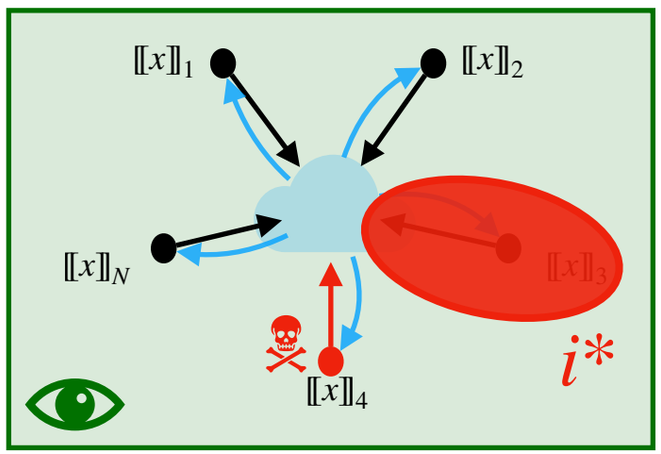
① Generate and commit shares

$$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$$

We have $F(x) \neq y$ where

$$x := [\![x]\!]_1 + \ldots + [\![x]\!]_N$$

② Run MPC in their head



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send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

③ Choose a random party

$i^* \leftarrow^{\$} \{1,\ldots,N\}$

⑤ Check $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $\tilde{g}(y, \alpha) =$ Accept

Malicious Prover

Verifier

Cheating detected!

MPCitH transform

① Generate and commit shares

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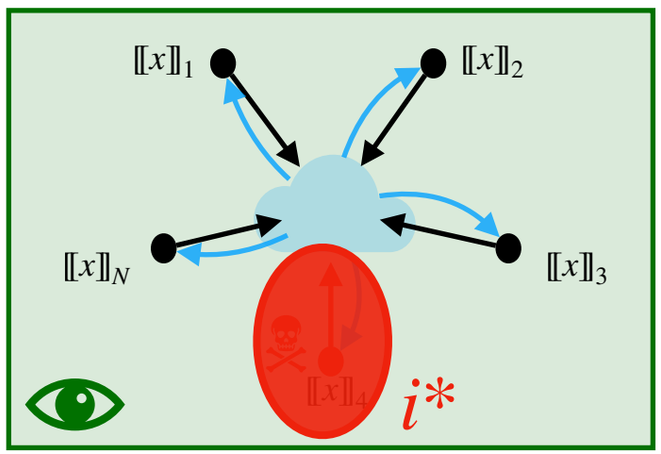

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Check $\tilde{g}(y, \alpha) =$ Accept

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

→

Malicious Prover

Verifier

Seems OK.

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private

MPCitH transform

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- **Soundness:**

$$\begin{aligned}&\mathbb{P}(\text{malicious prover convinces the verifier})\\ &\qquad = \mathbb{P}(\text{corrupted party remains hidden})\\ &\qquad = \frac{1}{N}\end{aligned}$$

MPCitH transform

- **Zero-knowledge** $\iff$ MPC protocol is $(N-1)$-private
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- **Parallel repetition**

Protocol repeated τ times in parallel → soundness error $\left(\frac{1}{N}\right)^{\tau}$

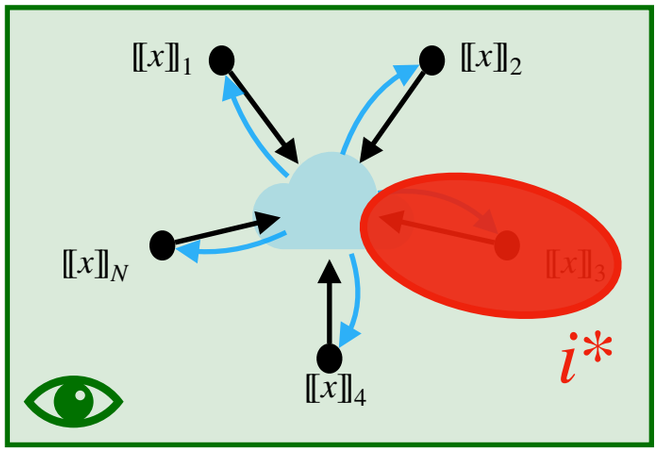
Optimisations

MPCitH transform

① Generate and commit shares

$$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$$

② Run MPC in their head



④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

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Check $\tilde{g}(y, \alpha) =$ Accept

Prover

Verifier

Naive MPCitH transformation

Size of the broadcast (per party)

Size of a commitment digest

Size of the MPC input (per party)

$$\text{Size} \approx \tau \cdot \left(N \cdot 2\lambda + N \cdot |\alpha| + (N-1) \cdot |x| \right)$$

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

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The signature sizes would be of at least 30 KB.

MPCitH transform

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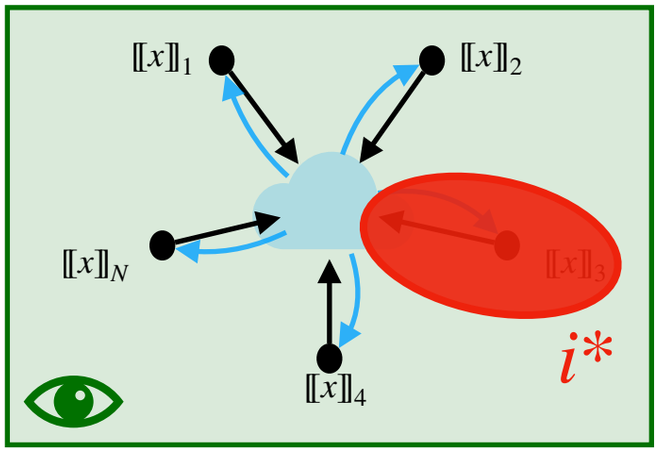
② Run MPC in their head



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$([\![x]\!]_i, \rho_i)_{i \neq i^*}$

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Prover

Verifier

MPCitH transform

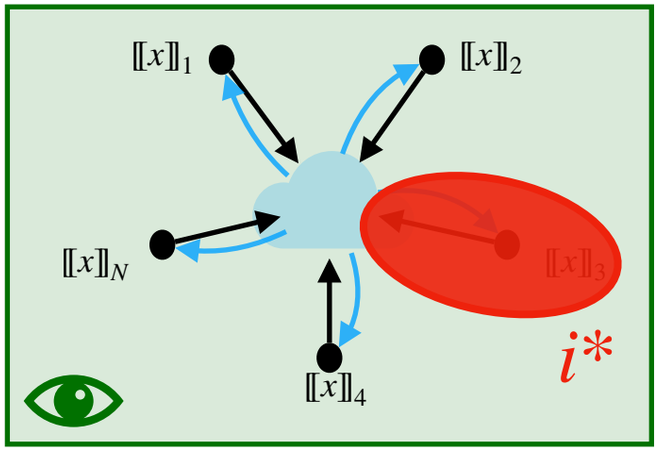
① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

Compute

$\forall i, \mathsf{com}_i = \mathrm{Com}^{\rho_i}([\![x]\!]_i)$

② Run MPC in their head



④ Open parties $\{1,\ldots,N\}\backslash\{i^*\}$

$h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$ →

$h_2 = \mathrm{Hash}([\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N)$ →

③ Choose a random party

$i^* \leftarrow^{\$} \{1,\ldots,N\}$

← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ $\quad (\mathsf{com}_{i^*}, \alpha)$ →

⑤ Compute $\forall i \neq i^*$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
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Check $h_1 = \mathrm{Hash}(\mathsf{com}_1, \ldots, \mathsf{com}_N)$

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Prover

Verifier

MPCitH transform

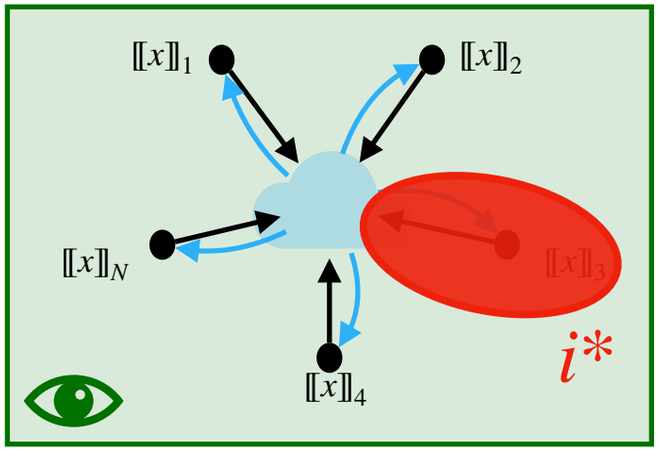
① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

Compute

$\forall i, \mathsf{com}_i = \mathrm{Com}^{\rho_i}([\![x]\!]_i)$

② Run MPC in their head



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← i^*

$([\![x]\!]_i, \rho_i)_{i \neq i^*}$ $(\mathsf{com}_{i^*}, \alpha)$ →

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Prover

Verifier

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \ldots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Using a Seed Tree

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$$x = \underset{\text{PRG}}{\overset{\text{seed}_1}{[\![x]\!]_1}} + \underset{\text{PRG}}{\overset{\text{seed}_2}{[\![x]\!]_2}} + \underset{\text{PRG}}{\overset{\text{seed}_3}{[\![x]\!]_3}} + \ldots + \underset{\text{PRG}}{\overset{\text{seed}_{N-1}}{[\![x]\!]_{N-1}}} + \underset{\text{PRG}}{\overset{\text{seed}_N}{[\![x]\!]_N}} + \Delta x$$

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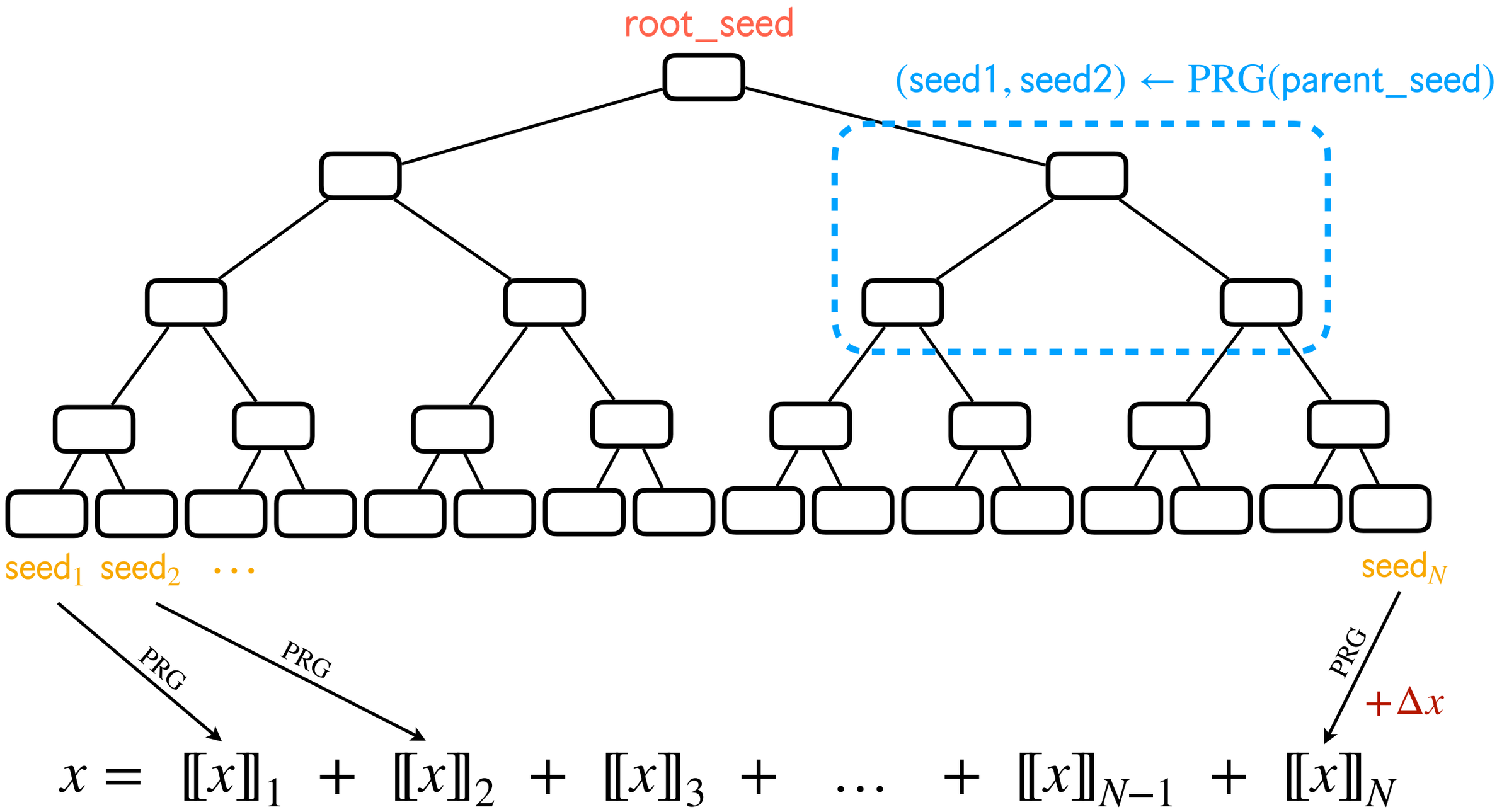

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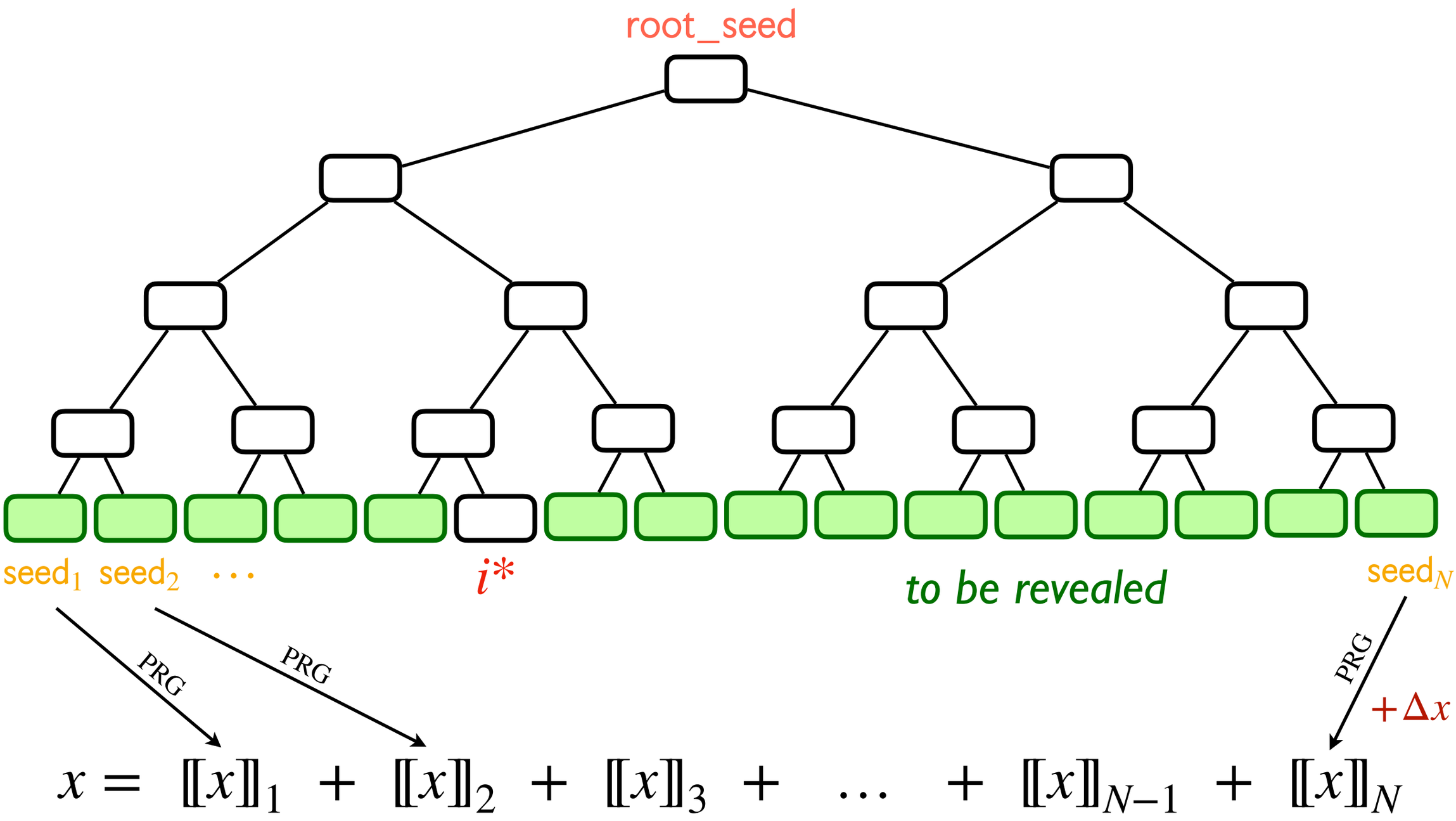

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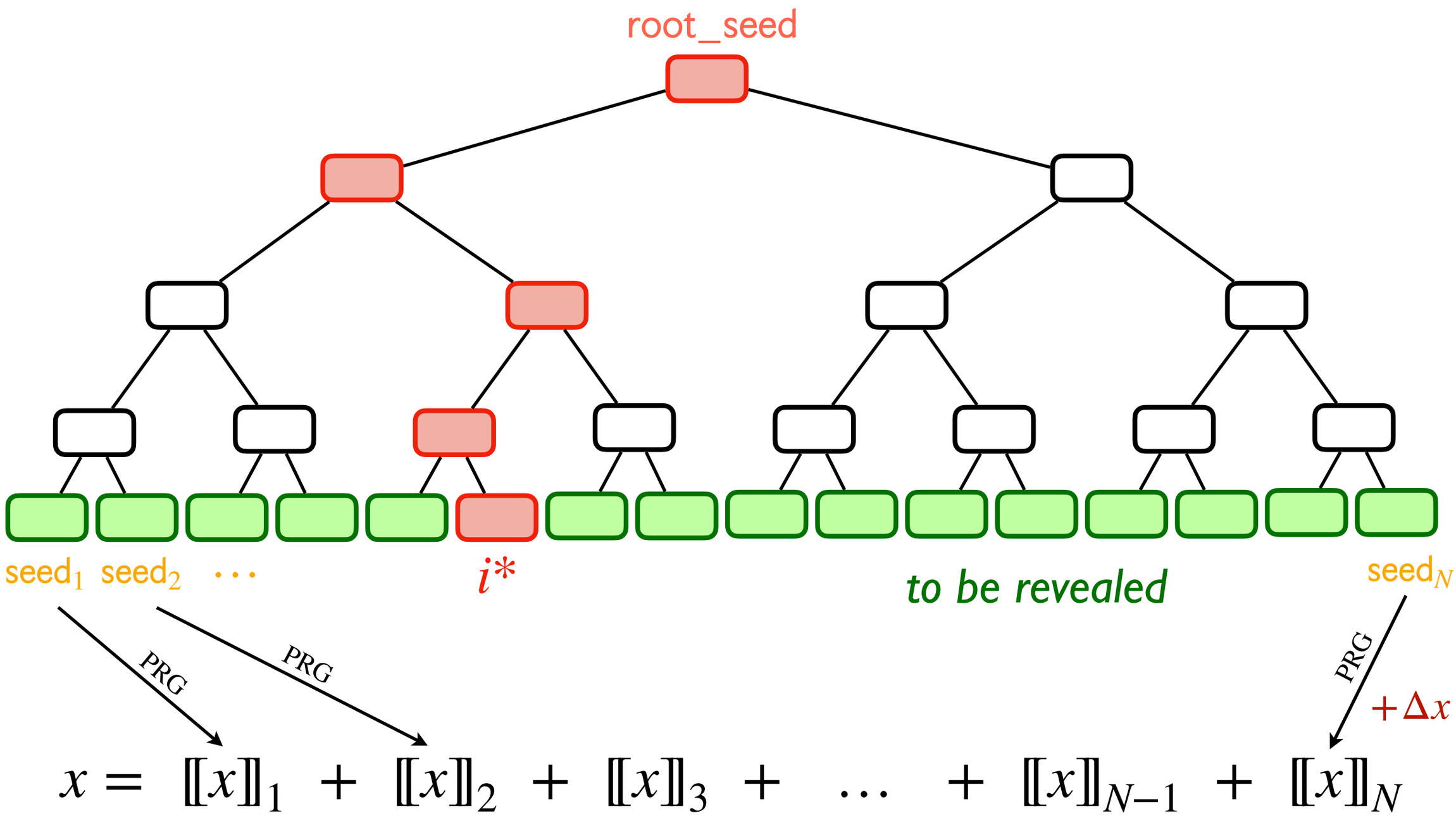

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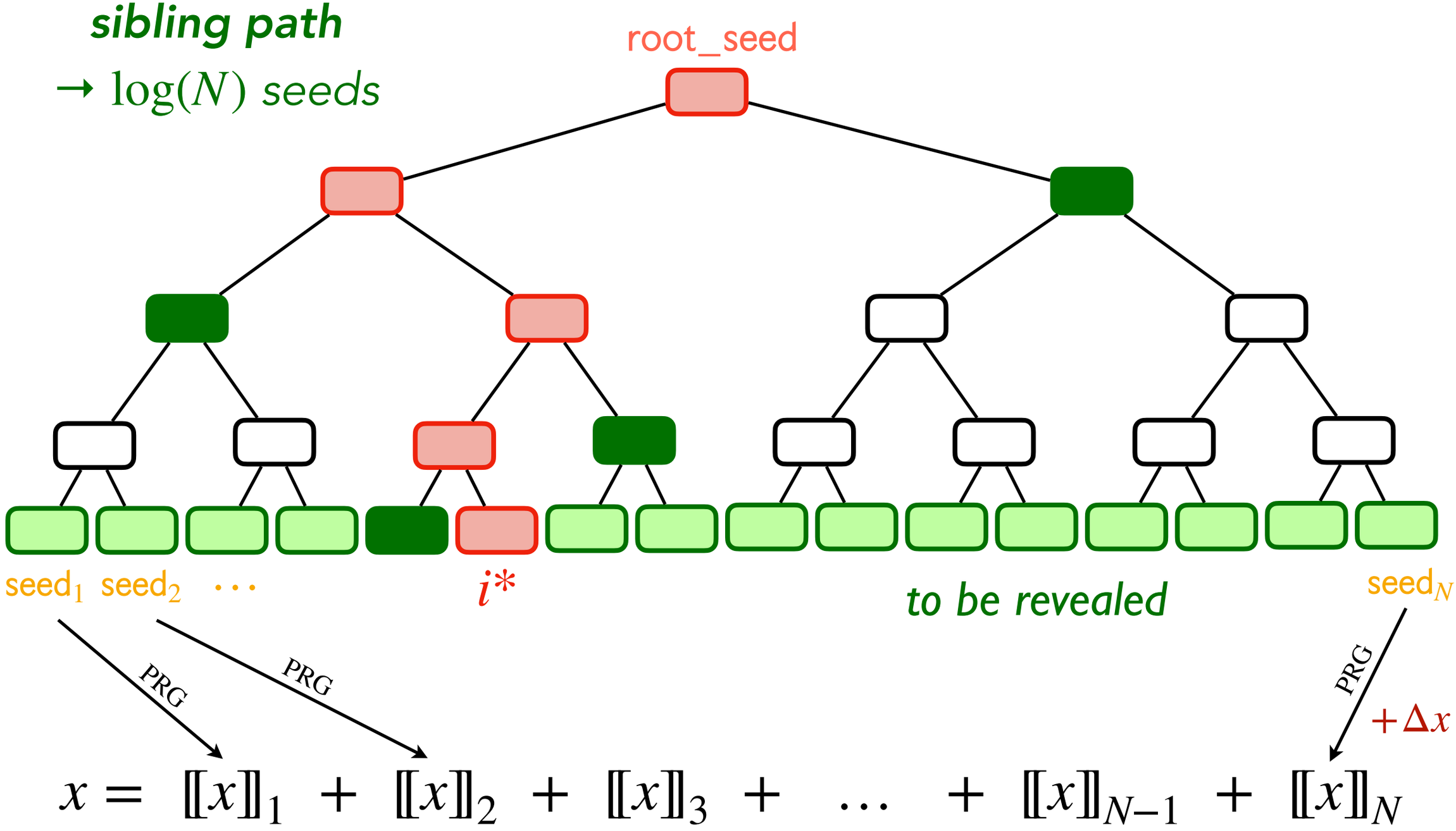
sibling path

→ $\log(N)$ *seeds*



$$x = [\![x]\!]_1 + [\![x]\!]_2 + [\![x]\!]_3 + \cdots + [\![x]\!]_{N-1} + [\![x]\!]_N$$

Traditional MPCitH transformation

Size of the broadcast

Size of the auxiliary value

Path in the seed (GGM) tree

$$\text{Size} \approx \tau \cdot \left(|\Delta x| + |\alpha| + \lambda \cdot \log_2 N + 2\lambda \right)$$

Commitment of the hidden party

Number of repetitions to achieve the desired security level

$$\tau \approx \frac{\lambda}{\log_2 N}$$

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Emulating τ MPC protocols with N parties is very expensive.

The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (Eurocrypt 2023)

Traditional: one sharing of x

$$x = r_1 + r_2 + \ldots + r_N + \Delta x$$

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Hypercube: D sharings of x, with the same auxiliary value Δx

$$x = \left\{ \begin{array}{c} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{array} \right\} + \Delta x$$

such that $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

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$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

How to build these D sharings?

The Hypercube Technique

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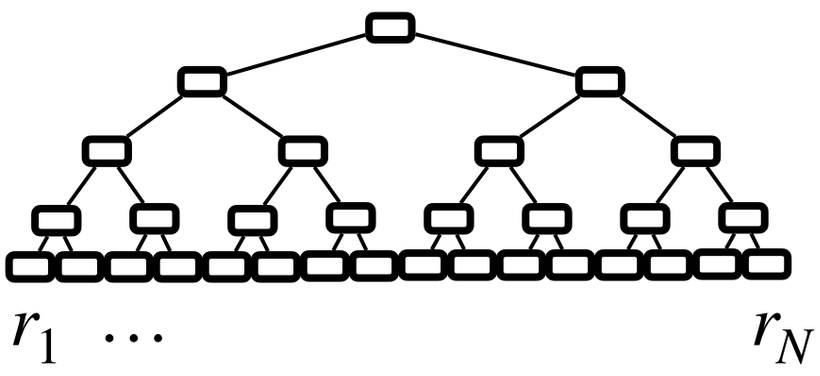

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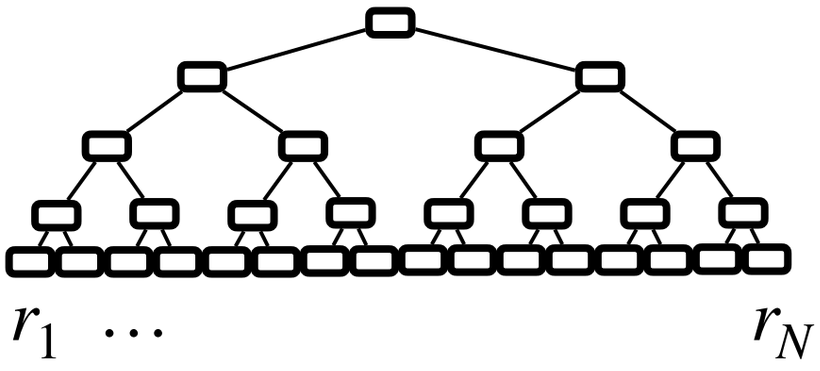
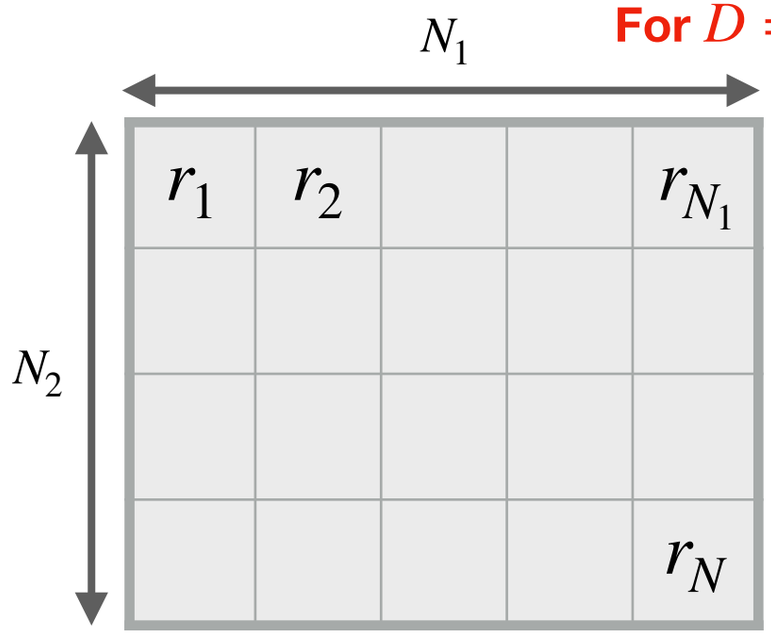


For $D = 2$



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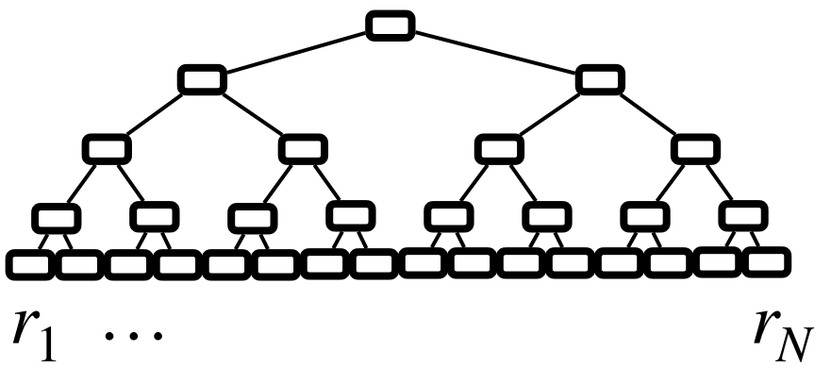

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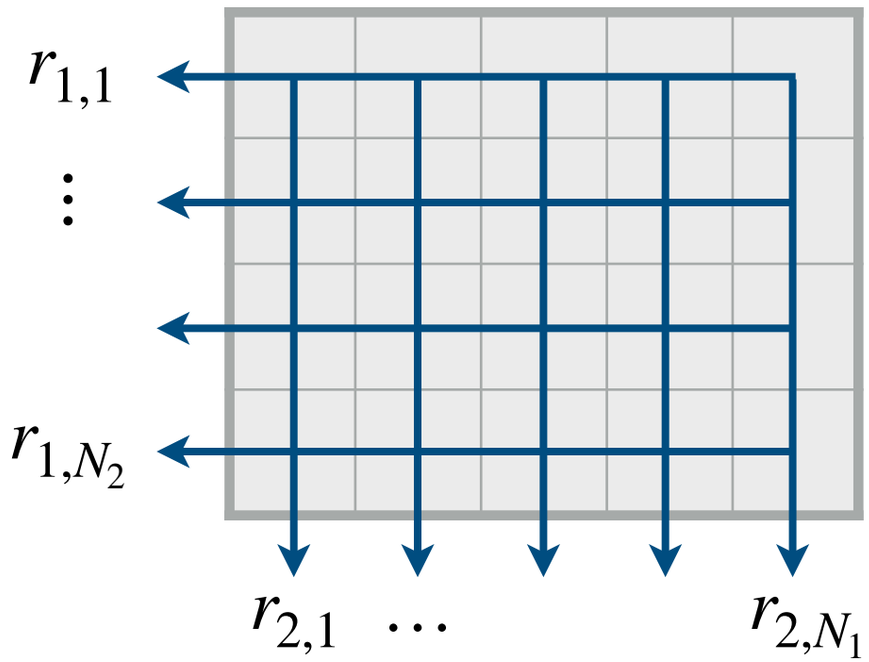

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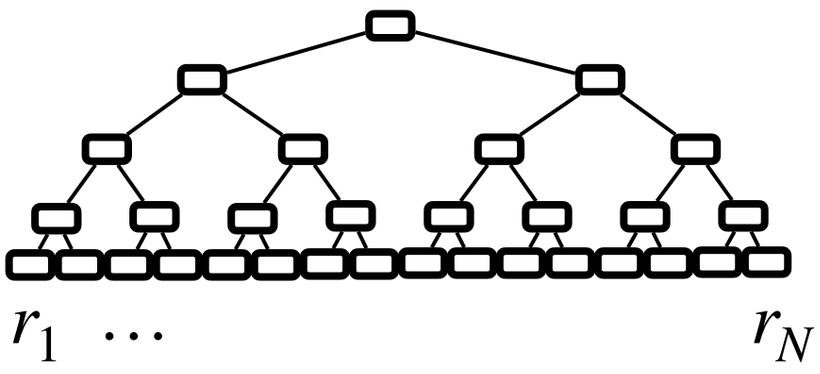
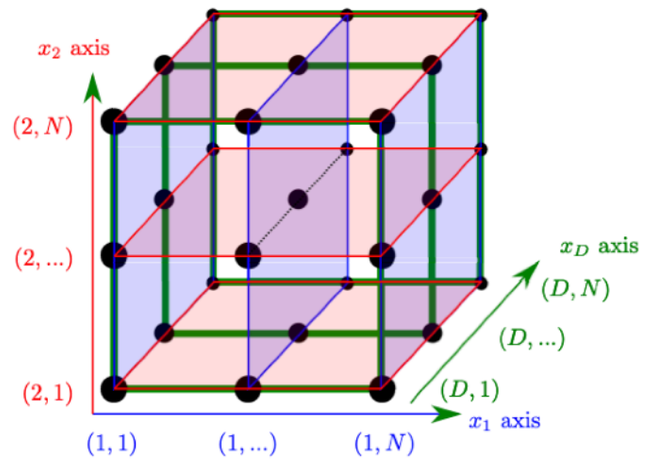
How to build these D sharings?



For $D \geq 2$



The $D \times N$ main party slices

Source: Figure from [AGHHJY23]

The Hypercube Technique

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$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

What about the soundness?

We emulate D sub-protocols…

$$\frac{1}{N_1} \cdot \frac{1}{N_2} \cdot \ldots \cdot \frac{1}{N_D} = \frac{1}{N}$$

Same soundness error as before!

The Hypercube Technique

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$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

What about the signature size?

$$\text{Size} \approx \tau \cdot \left(|\Delta x| + |\alpha| + \lambda \cdot \log_2 N + 2\lambda \right)$$

Same signature size as before!

The Hypercube Technique

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What about the emulation cost?

We emulate D sub-protocols…

$$N_1 + N_2 + \ldots + N_D$$

The Hypercube Technique

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The Hypercube Technique

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$$x = \left\{ \begin{array}{l} r_{1,1} + r_{1,2} + \ldots + r_{1,N_1} \\ r_{2,1} + r_{2,2} + \ldots + r_{2,N_2} \\ \ldots \\ r_{D,1} + r_{D,2} + \ldots + r_{D,N_D} \end{array} \right\} + \Delta x$$

$$N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$$

What about the emulation cost?

We emulate D sub-protocols…

~~$N_1 + N_2 + \ldots + N_D$~~

$$1 + (N_1 - 1) + (N_2 - 1) + \ldots + (N_D - 1)$$

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instead of $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

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$D = \log_2 N$

$N_1 = \ldots = N_D = 2$

$$1 + \log_2 N$$

instead of $N = N_1 \cdot N_2 \cdot \ldots \cdot N_D$

The Hypercube Technique

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Traditional: N party emulations per repetition

$N = 256$

$D = \log_2 N$

$N_1 = \ldots = N_D = 2$

Hypercube: $1 + \log_2 N$ party emulations per repetition

$1 + \log_2 N = 9$

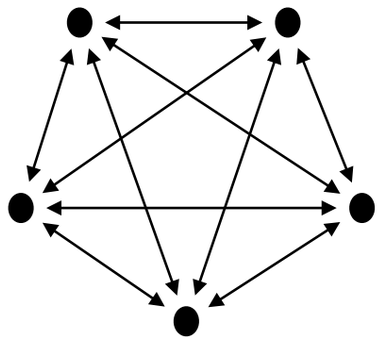
From MPC-in-the-Head to signatures

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Multiparty computation (MPC)

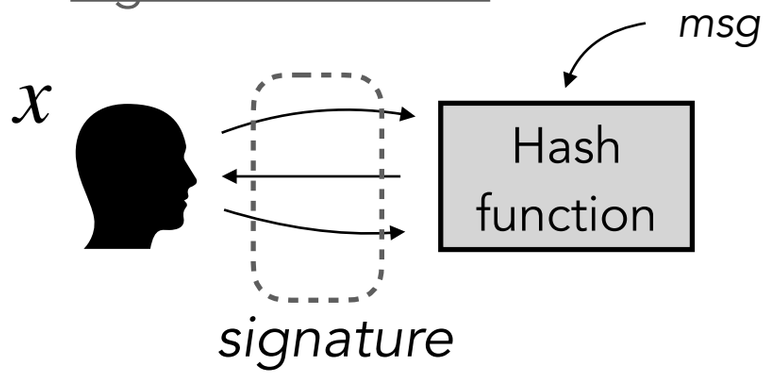


Input sharing $[\![x]\!]$

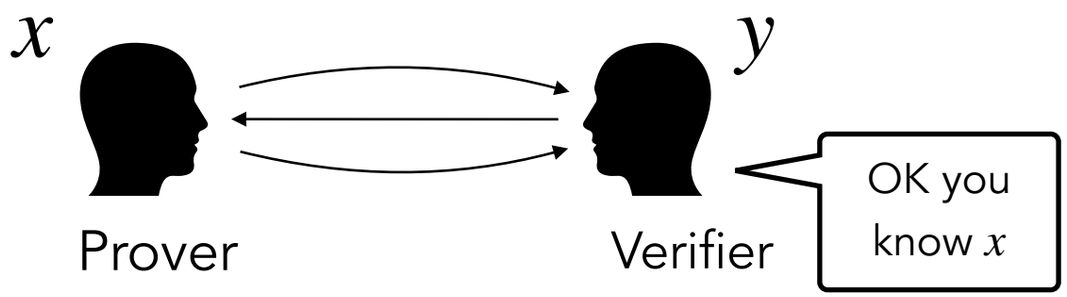
Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Signature scheme



Zero-knowledge proof



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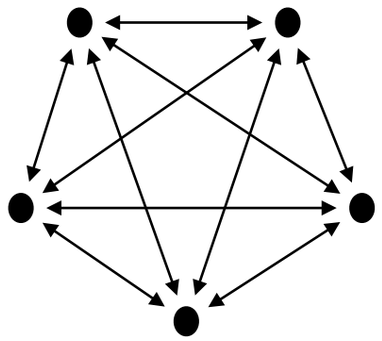

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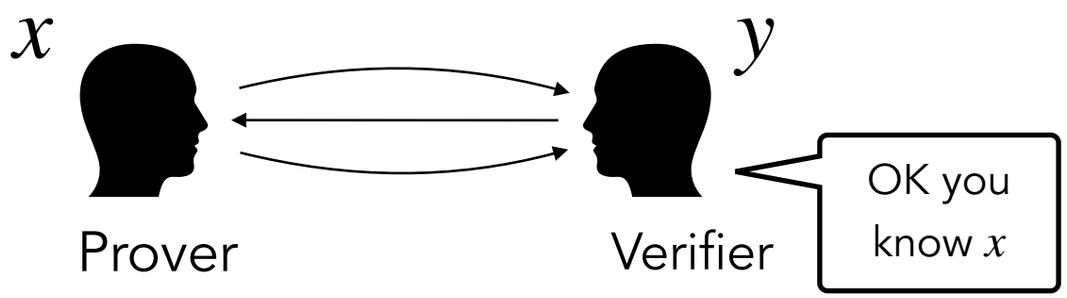
MPC-in-the Head transform

Zero-knowledge proof



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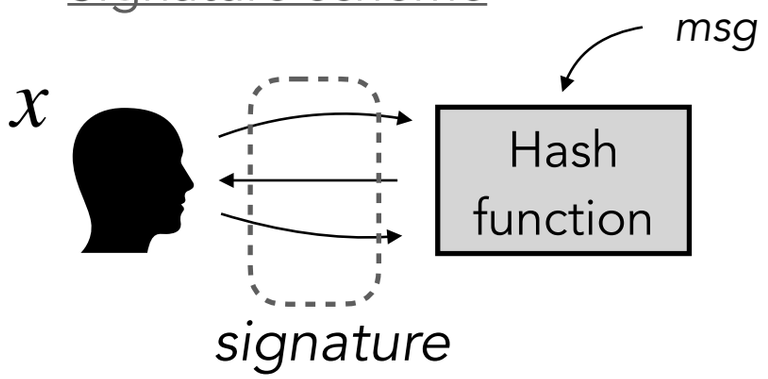

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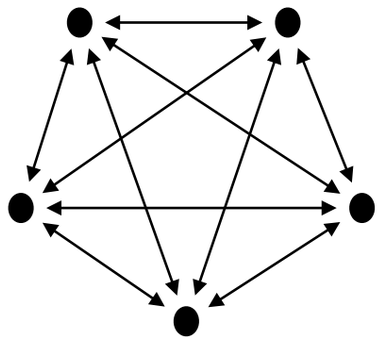

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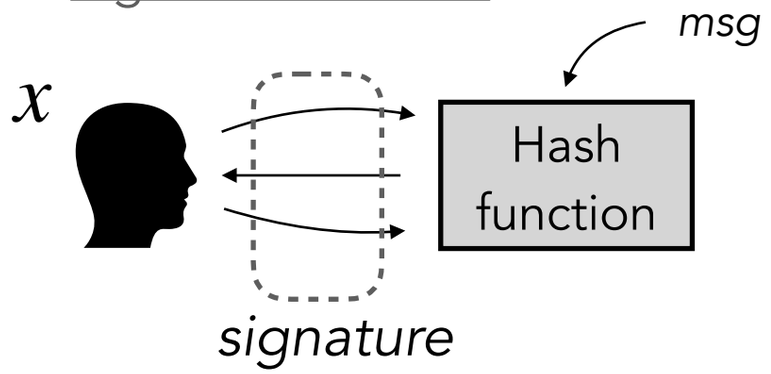

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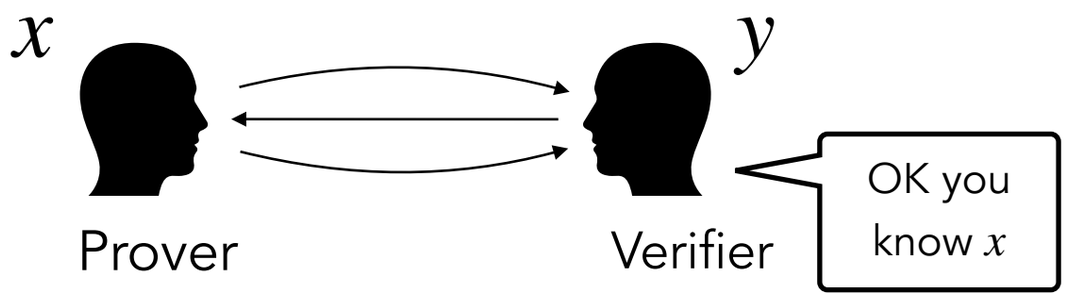

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- RYDE: Syndrome decoding problem in the rank metric

  From a matrix $H \in \mathbb{F}_{q^m}^{(n-k)\times n}$ and a vector $y \in \mathbb{F}_{q^m}^{n-k}$, find a vector $x \in \mathbb{F}_{q^m}^{n}$ s.t.

$$y = Hx \quad \text{and} \quad \dim \mathrm{Span}_{\mathbb{F}_q}(x_1, \ldots, x_n) \leq r.$$

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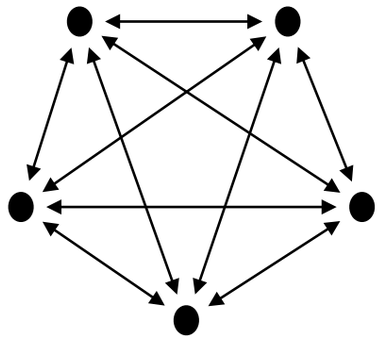

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The case of RYDE

Rank Syndrome Decoding Problem

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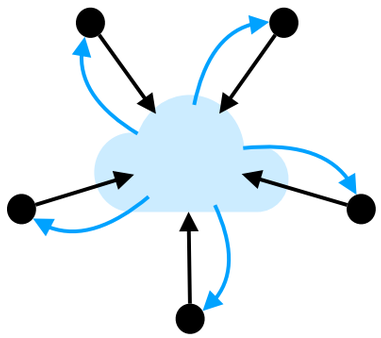

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- Check the linear constraint
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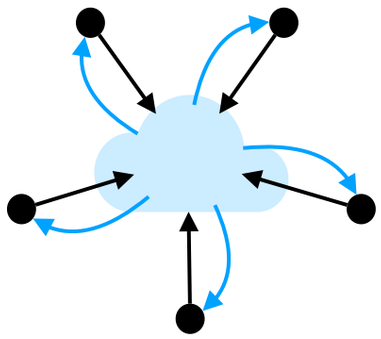

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and broadcast $[\![Hx]\!]_i$.

- Everyone can check that $[\![Hx]\!]_1 + \ldots + [\![Hx]\!]_N$ is equal to y.

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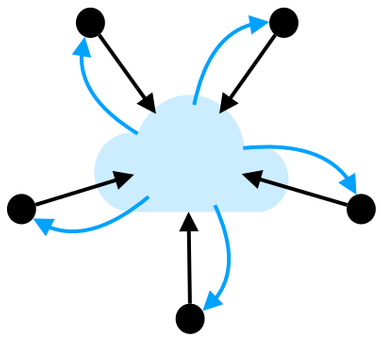

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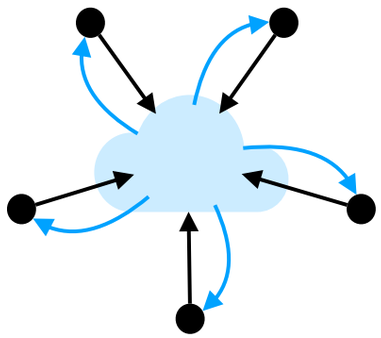

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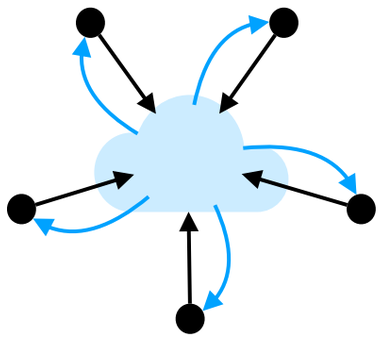

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[Fen22] Feneuil: "Building MPCitH-based Signatures from MQ, MinRank, Rank SD and PKP" (ePrint 2022/1512)

$$\dim \mathrm{Span}_{\mathbb{F}_q}(x_1, \ldots, x_n) \leq r$$

$$\Longleftrightarrow$$

$$\exists \text{ degree-}q^r\ q\text{-polynomial } P := \sum_{i=0}^{r} a_i X^{q^i} \text{ s.t. } L(x_1) = \ldots = L(x_n) = 0$$

The case of RYDE

Rank Syndrome Decoding Problem

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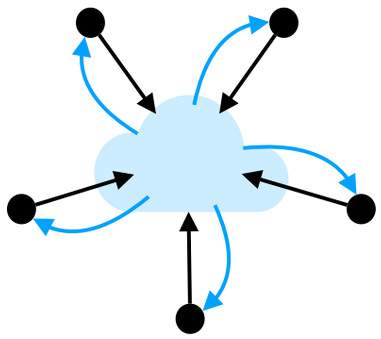

Input sharings $[\![x]\!]$ and $[\![P]\!]$

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- *Input of the MPC protocol*: two sharings $[\![x]\!]$ and $[\![P]\!] := \sum_{i=0}^{r} [\![a_i]\!] X^{q^i}$.

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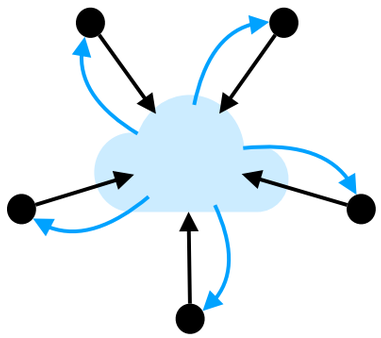

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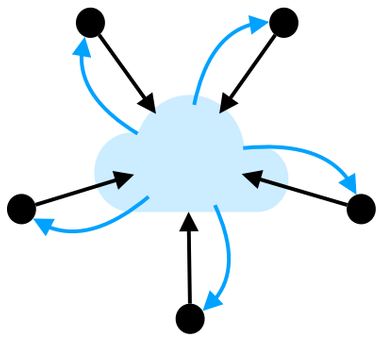

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- The parties will compute $[\![x_j^{q^i}]\!]$ from $[\![x_j]\!]$ for all (i, j).
- For all j, the parties will check that $P(x_j) = 0$ by checking that

$$\left\langle \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_r \end{pmatrix}, \begin{pmatrix} x_j \\ x_j^q \\ \vdots \\ x_j^{q^r} \end{pmatrix} \right\rangle = 0,$$

using a [BN20]-like MPC protocol.

[BN20] Baum, Nof: "Concretely-efficient zero-knowledge arguments for arithmetic circuits and their application to lattice-based cryptography" (PKC 2020)

The case of RYDE

Rank Syndrome Decoding Problem

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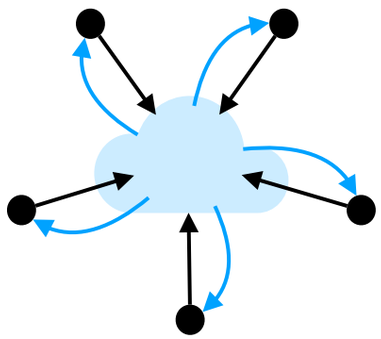

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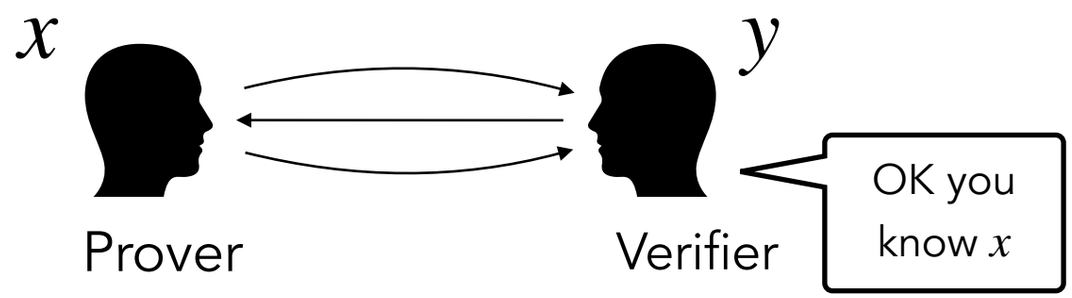
Zero-knowledge proof



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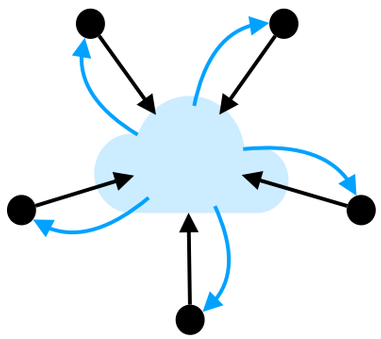

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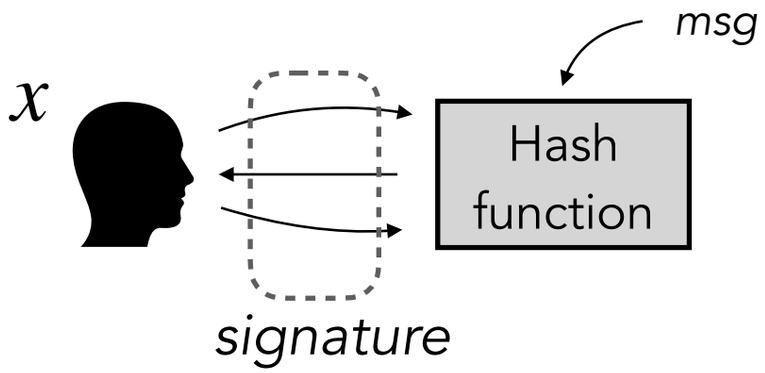

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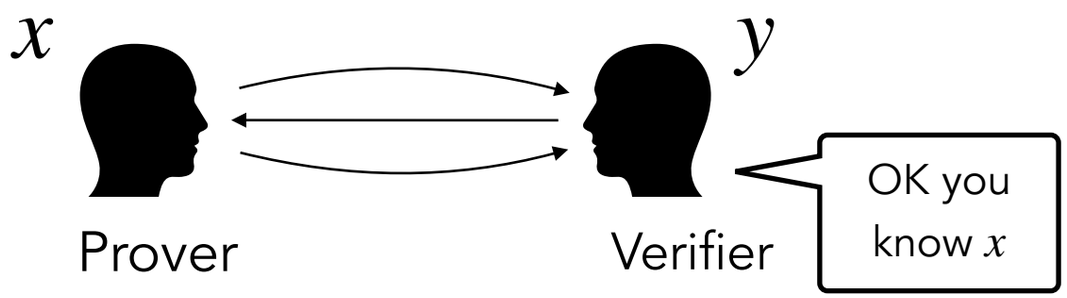

Fiat-Shamir transform

Should take [KZ20] attack into account (since there are 5 rounds)!

[KZ20] Kales, Zaverucha. "An attack on some signature schemes constructed from five-pass identification schemes" (CANS20)

The case of MIRA

MinRank Problem

From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

$$\operatorname{rank}\left(M_0 + \sum_{i=1}^{k} x_i M_i\right) \leq r.$$

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From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

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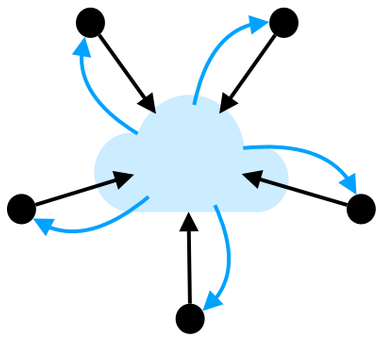

Input sharing $[\![x]\!]$

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- *Input of the MPC protocol*: a sharing $[\![x]\!]$ of x.

- Let us denote $E = M_0 + \sum_{i=1}^{k} x_i M_i \in \mathbb{F}_q^{m \times n}$.

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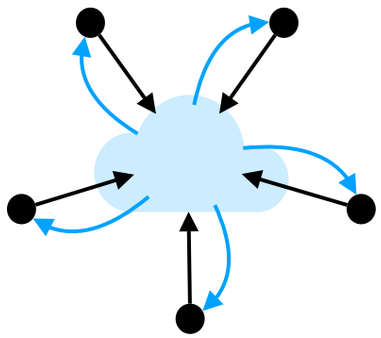
Multiparty computation (MPC)



Input sharing $[\![x]\!]$

- Check the rank constraint

- *Input of the MPC protocol*: a sharing $[\![x]\!]$ of x.

- Let us denote $E = M_0 + \sum_{i=1}^{k} x_i M_i \in \mathbb{F}_q^{m \times n}$.

- We decompose $E = \left(E_1 \mid E_2 \mid \ldots \mid E_n\right)$ by columns and write each column E_i as a field element $e_i \in \mathbb{F}_{q^m}$. We thus have

$$\operatorname{rank}(E) \leq r \iff \dim \operatorname{Span}_{\mathbb{F}_q}(e_1, \ldots, e_n) \leq r.$$

The case of MIRA

MinRank Problem

From $M_0, \ldots, M_k \in \mathbb{F}_q^{m \times n}$, find x such that

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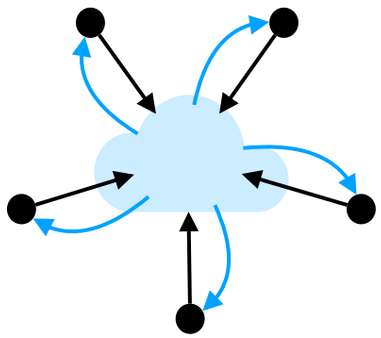
Multiparty computation (MPC)



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- We can use the *same protocol* than with RYDE.

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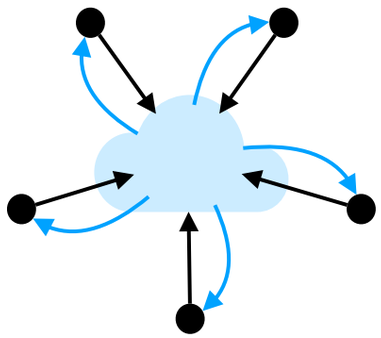

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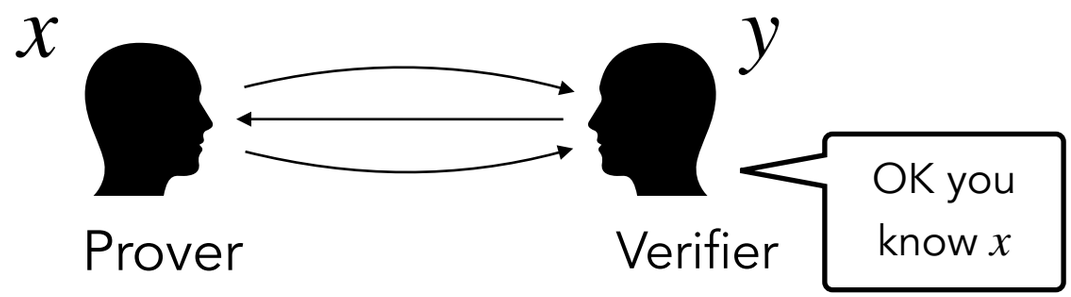
Zero-knowledge proof



The case of MIRA

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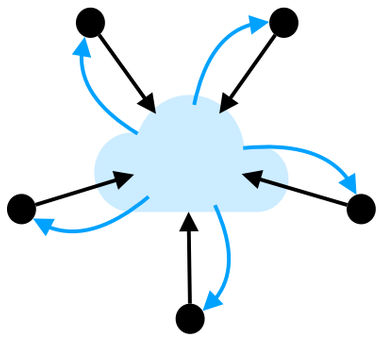

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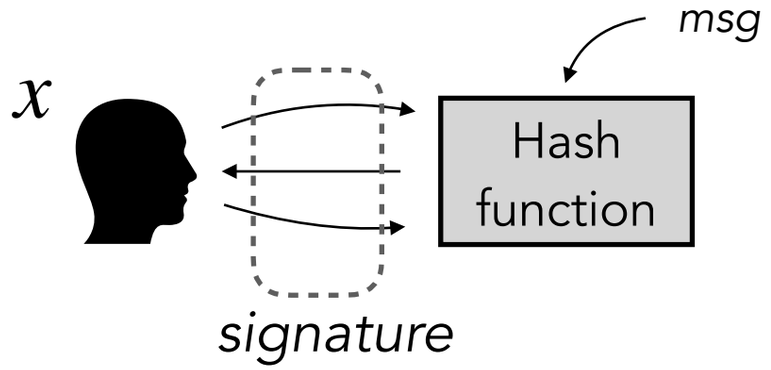
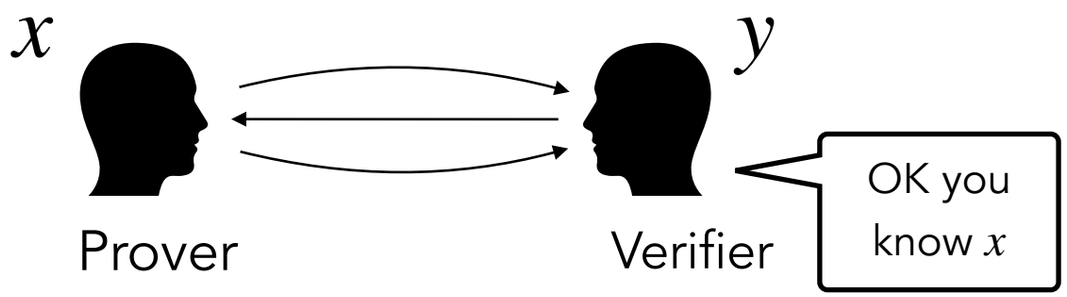
MIRA



Zero-knowledge proof



Performances

Performances

	Short Instance			Fast Instance		
	$\|\text{sig}\|$	t_{sign}	t_{verify}	$\|\text{sig}\|$	t_{sign}	t_{verify}
RYDE L1	6.0	23.4	20.1	7.4	5.4	4.4
MIRA L1	5.6	46.8	43.9	7.4	37.4	36.7
RYDE L3	12.9	49.6	44.8	16.4	12.2	10.7
MIRA L3	11.8	119.7	116.2	15.5	107.2	107.0
RYDE L5	22.8	105.5	94.9	29.1	26.0	22.7
MIRA L5	20.8	337.7	331.4	27.7	322.3	323.2

All public keys are smaller than 200 bytes.

Isochronous implementations
Size in kilobytes, timing in Mcycles
@2.60GHz: 1 millisecond ≈ 2.6 Mcycles

Advantages and limitations

- Limitations
  - Relatively ***slow*** *(few milliseconds)*
    - Greedy use of symmetric cryptography
  - Relatively ***large*** signatures (*5.5-7.5 KB for L1*)
  - Signature size: ***quadratic*** growth in the security level

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- Advantages
  - ***Conservative*** hardness assumption:
    - No structure, no trapdoor
  - ***Small*** (public) keys
  - ***Good*** public key + signature size
  - Adaptive and ***tunable*** parameters

Rank Metric in the Head

RYDE

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn, T. Feneuil, P. Gaborit, A. Joux, M. Rivain, J.-P. Tillich, A. Vincotte

`https://pqc-ryde.org`

MIRA

N. Aragon, M. Bardet, L. Bidoux, J.-J. Chi-Domínguez, V. Dyseryn, T. Feneuil, P. Gaborit, R. Neveu, M. Rivain, J.-P. Tillich

`https://pqc-mira.org`

Thank you for your attention.