SOLUTIONS FOR ADMISSIONS TEST IN
MATHEMATICS, COMPUTER SCIENCE AND JOINT SCHOOLS
OCTOBER 2025

Question 26 X

(i) There are eight points and we choose any two, so there are $(8 \times 7)/2$ pairs, which is 28. But we do not want the four lines that connect points directly opposite each other. So 24 lines.

(ii) Choose the first point (8 choices), then choose one of the 6 points that is not opposite, then choose one of the 4 points that's not opposite either of the first two. This generates each 3-star exactly six times, once for each ordering of the points. So the total is $(8 \times 6 \times 4)/6 = 32$.

(iii) For a 4-star, we'd need one point from each of the directly-opposite pairs. So $2 \times 2 \times 2 \times 2$ choices, which is 16.

There are no 5-stars or 6-stars or 7-stars or 8-stars, because these would need at least one pair of directly-opposite points.

(iv) A 3-star can't have two points in A or two points in B or two points in C (because those two points would not be connected), so it must have one in each set. Then any choice works, because every point in A is connected to each point in B, and each point in B is conenected to each point in C, and each point in C is connected to each point in A. So we have free choice from each set, for a total of $3 \times 4 \times 5 = 60$ 3-stars.

(v) We can form a 4-loop by choosing points in sets in the following order (choosing a new point from the set each time it comes up); CBCB. There are lots of other solutions.

The rules for these sequences are that we can't have two adjacent letters the same, and we can't use A more than 3 times, B more than 4 times, or C more than 5 times. Here are examples of loops of each length from 5 to 12;

5-loop: CBCBA

6-loop: CBCBCB

7-loop: CBCBCBA

8-loop: CBCBCBCB

9-loop: CBCBCBCBA

10-loop: CBCBCBCBCA

11-loop: CBCBCBACBCA

12-loop: CBCBACBACBCA

We definitely can't have more than 12 points in our n-loop, because there are only 12 points overall. So the possible values of n are all the whole numbers from 4 to 12 inclusive.

Question 26 Y

(i) (a) $(a + bx) \cdot (c + dx) = ac + (ad + bc)x$. If we did this in the other order, we'd get $ca + (cb + da)x$, which is the same thing. So $f(x) \cdot g(x) = g(x) \cdot f(x)$.

(b) If we have $a = 1$ and $b = 2$ then to get $3 + 4x$ for the product, we want $c = 3$ and $d + 2c = 4$, so $c = 3$ and $d = -2$. Thus $f(x) = 3 - 2x$.

(ii) The student is not correct. There are many counter-examples, such as

$$(1 + x) \cdot (-2 + x) = -2 - x,$$

where $1 + x$ and $-2 + x$ are strictly increasing linear functions, but $-2 - x$ is not.

(iii) Let the polynomials be $a + bx$ and $c + dx$ and $p + qx$. Then

$$f(x) \cdot g(x) = ac + (ad + bc)x$$

as before, and

$$(f(x) \cdot g(x)) \cdot h(x) = acp + (ac(f) + (ad + bc)p)x = acp + (acq + adp + bcp)x.$$

Combining polynomials in the other order amounts to switching a with p, and switching b with q, in the calculation above. So we'll get

$$f(x) \cdot (g(x) \cdot h(x)) = pca + (pcb + pda + qca)x,$$

which is the same linear polynomial.

(iv) $(1+x) \cdot (1+2x) = 1+(1+2)x$ and then $(1+(1+2)x) \cdot (1+3x) = 1+(1+2+3)x$ and this pattern continues. We want $1 + (1 + 2 + 3 + \cdots + 2N)x$. That second coefficient is the sum of the terms of an arithmetic progression, and the sum is $2N(2N + 1)/2 = N(2N + 1)$. So the answer is $1 + N(2N + 1)x$.

(v) Let $f(x) = a + bx$ and $g(x) = c + dx$. We must have $ac = 0$ and $ad + bc = 0$. So either $a = 0$ or $c = 0$.

- If $a = 0$ then $bc = 0$. So either $b = 0$ (and so $f(x) = 0$) or $c = 0$.
- And vice versa; if $c = 0$ then either $g(x) = 0$ or $a = 0$.

Therefore there are three possibilities; $f(x) = 0$, or $g(x) = 0$, or f and g are of the form bx and dx for some real b and d.

(vi) Let $F(x) = f(x) \cdot g(x)$. Then by the previous part, either $h(x) = 0$ (in which both case II and III hold) or $F(x) = 0$ (i.e. I holds), or $F(x) = bx$ and $h(x) = dx$. Now if $f(x) \cdot g(x) = bx$, note that the constant coefficient is zero, so one or the other of $f(x)$ and $g(x)$ has zero constant coefficient. Therefore that polynomial, whichever it is, $\cdot$ with $h(x)$ will be zero (i.e. either II or III holds).

For an example where (I) is true and (II) and (III) are both false, take $f(x) = x$ and $g(x) = x$ and $h(x) = 1$.

Similarly for the other two cases, taking a different one of the polynomials to be 1 and the others to be x.

Question 27 X

(i) No. The smallest has to be paired with largest (else the largest plus x would surely be larger than y plus the smallest). So the target needs to be 12. But there's nothing to pair with 4 (or 5, or 6, or 9) to make that target.

(ii) The six numbers can be paired up such that each pair sums to T. Adding them all together will therefore give $T + T + T = 3T$. That's a multiple of 3.

(iii) Example with zero targets; the strictly positive whole numbers. Suppose T were the target, but then there's nothing to pair with (for example) T itself.

Example with exactly one target; the whole numbers, positive or negative, but don't include zero. This has target 0 because we can pair x with $-x$ for each x. There can't be any other value of the target T because we wouldn't be able to pair anything with T itself.

Example with more than one target; all the whole numbers, positive or negative. We can have $T = 1$ by pairing x with $1-x$. We could instead have $T = 3$ by pairing x with $3-x$. No element is paired with itself under either of these plans (because in the first case $x = 1 - x$ is only satisfied by $x = \frac{1}{2}$, and in the second case $x = 3 - x$ is only satisfied by $x = \frac{3}{2}$, and neither of these values is a whole number).

There are many other examples of each.

(iv) Two of them are the same colour (they can't all be different colours), and any two numbers x and y always form a nice set with target $x + y$.

(v) Consider the 9 sets in the hint. There are only eight ways to colour three ordered items; RRR / RRG / RGR / RGG / GRR / GRG / GGR / GGG, so two of these nine sets must be coloured in the same way.

Within each of those two matching sets, there are two elements that are the same colour. Call the elements in the first set x and $x+A$, and the elements in the second set y and $y + A$, where A is either 1 or 2, but it's the same for the two matching sets.

These four numbers form a nice set with target $x + y + A$, because $x + (y + A)$ is $(x + A) + y$.

Question 27 Y

(i) The list (2,1,1) means that the first team books room 2, then the second team books room 1, and then the third team books room 3 (the third team is just one person, but it's OK for them to take a room that's larger than the team).

The list (2,2,2) has nobody willing to take room 1, so it's not good. Alternatively, note that the first team will be given room 2, then the second team will be given room 3, and then the third team won't fit in room 1.

(ii) (1,1,1) (1,1,2) (1,1,3) (1,2,1) (1,2,2) (1,2,3) (1,3,1) (1,3,2)

(2,1,1) (2,1,2) (2,1,3) (2,2,1) (2,3,1)

(3,1,1) (3,1,2) (3,2,1)

(iii) $F(n, k)$ means that the last team gets meeting room k (other rooms have been booked). They have to fit into that meeting room with space for k people, so their team size might be 1 or 2 or ... or k. So the good lists we're counting can be split into sets that are identical except for the last number, with (say) m items in each set. So we have $F(n, k) = m \times k$, and therefore the total $F(n, k)$ is a multiple of k.

(iv) $G(n) = F(n, 1) + F(n, 2) + \cdots + F(n, n)$.

(v) $F(4, 1)$ means that there are four teams and the fourth team ended up in room 1. That tells us two things; that last team is a team of just one person, and it must be the case that no other team has exactly one person (else they'd be in room 1).

So the other three teams have shared out rooms 2, 3, and 4, and all of those teams have at least two people in them. This is the same problem as part (ii). To see this, take the lists from (ii) and add one to each room number and add one person to each team. There were $G(3) = 16$ solutions in part (ii). So $F(4, 1) = G(3) = 16$.

$F(4, 4)$ means that out of the first three teams, no team was so large that they had to go in room 4. So the first three teams have shared out rooms 1, 2, and 3. There are $G(3)$ ways to do that. Now the fourth team might have any number of people up to 4, independent of what's happened before, so there are $4 \times G(3) = 64$ good lists where the last team gets meeting room 4.

(vi) The situation for $F(4, 2)$ just before the fourth team calls reception is as follows; exactly one team had one person and got room 1 (three possibilities for which team it was), and the other two teams were too big for room 2 and have sorted out rooms 3 and 4 between them, with $G(2) = 3$ ways to do that. Then the last team has size either 1 or 2, so that they fit in room 2. Multiplying gives $3 \times 3 \times 2 = 18$ possibilities.

The situation for $F(4, 3)$ just before the fourth team calls reception is that exactly one team needed room 4 (not more than one team, or they would have been a problem when the second of them called). There are three possibilities for which team had size 4. The other teams sorted out rooms 1 and 2 between them, with $G(2) = 3$ ways to do that sorting out. Then the last team has to fit in meeting room 3, so they have size 1 or 2 or 3. Multiplying gives $3 \times 3 \times 3 = 27$ possibilities.

The total for $G(4)$ is $16 + 18 + 27 + 64 = 125$.