# Introduction to the Birch-Swinnerton-Dyer Conjecture III: Average ranks, the Artin-Tate conjecture and function fields.

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In the previous talks we have seen the formulation of the Birch-Swinnerton-Dyer conjecture. This talk will focus on a fundmental question in diophantine geometry. Namely, given an algebraic curve $C$ defined over $\mathbf{Q}$ possessing at least one rational point, what is the probability that $C$ has infinitely many rational points?

For curves of genus 0, the answer has been known ever since the ancient Greeks roamed the earth, and for genus $> 1$ the answer is also known (albeit for a much shorter time). The remaining case is genus 1, and this question has a history filled with tension and conflict between data and conjecture.

I shall describe the heuristics behind the conjectures, taking into account the Birch-Swinnerton-Dyer Conjecture and the Parity Conjecture. I shall go on to outline the contrary numeric data, both in families of elliptic curves and for all elliptic curves of increasing conductor.

If one instead considers elliptic curves over function fields $\mathbf{F}_q(t)$, then, via a conjecture of Artin and Tate, one can compute the rank (and more) of elliptic curves of extremely large discriminant degree. I shall briefly describe the interplay between Artin-Tate and Birch-Swinnerton-Dyer, and give new evidence finally supporting the conjecture.