Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics & Philosophy
Syllabus and Synopses for Part B 2009–2010
for examination in 2010

Contents

1 Foreword

This Supplement to the Mathematics Course Handbook specifies the Mathematics courses available for Part B in Mathematics & Philosophy in the 2010 examination. It should be read in conjunction with the Handbook for Mathematics & Philosophy for the academic year 2009–2010, to be issued in Michaelmas Term. The Handbook contains in particular information on the format and rubrics for written examination papers in Mathematics, and the classification rules applicable to Part B.

See the current edition of the *Examination Regulations* for the full regulations governing the examinations.

1.1 Part B of the Honour School of Mathematics & Philosophy

The following is reproduced from the *Examination Regulations* applicable to the 2010 examinations.

The examination for Part B shall consist of units in Mathematics and subjects in Philosophy. The schedule of units in *Mathematics* shall be published in a supplement to the Mathematics Course Handbook by the beginning of the Michaelmas Full Term in the academic year of the examination concerned. The schedule shall be in two parts: Schedule 1 (standard units and half-units) and Schedule 2 (additional units and half-units). In *Philosophy* the subjects shall be subjects 101–118, 120, 122, 124 and 199 from the list given in *Special Regulations for All Honour Schools Including Philosophy*. Each subject in Philosophy other than a Thesis shall be examined in one 3-hour paper. Each candidate shall offer

(i) Two units of *Mathematics* from Schedule 1, one of which shall be B1 *Foundations: Logic and Set Theory*,

(ii) three subjects in *Philosophy* from 101-118, 120, 122 and 124, of which two must be 122 and **either** 101 **or** 102, and

(iii) **either** one further unit in *Mathematics* drawn from Schedules 1 and 2 combined **or** one further subject in *Philosophy* from subjects 101-118, 120, 124, and 199: *Thesis*.

Further information on the units in Mathematics is given below.

1.2 "Units" and "half-units" and methods of examination

Most courses in Mathematics are assessed by examination. Most subjects offered have a 'weight' of one unit, and will be examined in a 3-hour examination paper. In many of these subjects it will also be possible to take the first half, or either half, of the subject as a 'half-unit'. Where this is the case, a half-unit will be examined in an examination paper of $1\frac{1}{2}$ hours duration. Each half-unit paper will contain **3** questions.

1.3 The Schedules of Mathematics units and half-units for Mathematics & Philosophy

All units and half-units in Mathematics are drawn from the list of options for Mathematics Part B.

Schedule 1 comprises those Mathematics Department courses for which the core and options in Mathematics & Philosophy Part A provide the requisite background.

Schedule 2 contains an Extended Essay option and certain further courses from Mathematics Part B appropriate for the Joint School. The latter include certain courses in pure mathematics from Mathematics Part C (M-level) which are designated as being available in the third year; they are examined using the same examination questions/projects as are set for fourth year students. Any particular course falling under this heading is normally only given in alternate years. Therefore certain courses available this year may not be offered for Part C in 2011. In addition you may apply for special approval to be examined in Mathematics Department units and half-units not included under Schedule 1; any such subject approved with be treated as falling under Schedule 2. For the procedure for seeking approval, see Subsection 1.4 below.

For the 2010 examination, the Schedules are as follows.

Schedule 1 (standard units and half-units)

B1 Foundations: Logic and Set Theory — whole unit

[Compulsory for Mathematics & Philosophy]

B2 Algebra — whole unit

- **B2a** Introduction to Representation Theory — half-unit
- **B2b** Group Theory — half-unit

B3 Geometry — whole unit

- **B3a** Geometry of Surfaces — half-unit
- **B3b** Algebraic Curves — half-unit

B4 Analysis — whole unit

- **B4a** Banach spaces — half-unit

B9 Number Theory — whole unit

- **B9a** Galois theory — half-unit

B10a Martingales through measure theory — half-unit

B11a Communication theory — half-unit

C3.1a Topology and Groups M-level — half-unit

Schedule 2 (additional units and half-units)

BE Mathematical Extended Essay		whole unit
O1 History of Mathematics		whole unit
OCS3a Lambda Calculus and Types		half-unit
N1 Undergraduate Ambassadors' Scheme		half-unit
C3.2 Lie Groups and Differentiable Manifolds	M-level	whole unit
C3.2a Lie Groups	M-level	half-unit
C3.2b Differentiable Manifolds	M-level	half-unit

And also

Any other whole unit or half-unit course from the list of Mathematics Department units, other than those in Schedule 1 and BE: Extended Essay, for which special approval has been granted.

1.4 Procedure for seeking approval of additional options where this is required

You may, if you have the support of your Mathematics tutor, apply to the Joint Committee for Mathematics and Philosophy for approval of one or more other options from the list of Mathematics Department units and half-units for Part B. This list can be found in the Supplement to the Mathematics Course Handbook giving syllabuses and synopses for courses in for Mathematics Part B.

Applications for special approval must be made through the candidate's college and sent to the Chairman of the Joint Committee for Mathematics and Philosophy, c/o Academic Administrator, Mathematical Institute, to arrive by **Friday of Week 5 of Michaelmas Term**. Be sure to consult your college tutors if you are considering asking for approval to offer one of these additional options.

Given that each of these additional options, which are all in applied mathematics, presume facility with some or other results and techniques covered in first or second year Mathematics courses not taken by Mathematics & Philosophy candidates, such applications will be exceptional. You should also be aware that there may be a clash of lectures for specially approved options and those listed in Schedules 1 and 2 and with lectures in Philosophy; see the section in The Mathematics Part B Synopses on lecture clashes.

1.5 Registration for Part B courses 2009–2010

CLASSES Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2009 using the online registration system which can be accessed at https://www.maths.ox.ac.uk/course-registration .

Students will need to login using OU Webauth. It will then be possible for students to select the units and half-units they wish to take from the drop-down menu. Further guidance on how to use the online system can be found at:
http://www.maths.ox.ac.uk/help/faqs/undergrads/course-registration.

Students who register for a course or courses for which there is a quota will have to select a "reserve choice" which they will take if they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor. Students who are not allocated a place on the course with the quota will be registered for their "reserve" course. Students will be notified of this by email. In the case of the "Undergraduate Ambassadors' Scheme" students will have to attend a short interview in Week 0, Michaelmas Term. In the meantime they will also have to complete a separate application form, and indicate a "reserve" course.

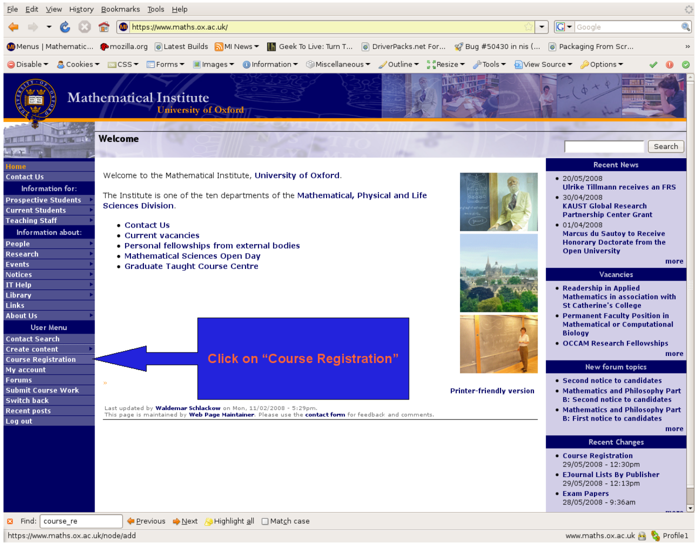

Figure 1: Logging on to the registration system.

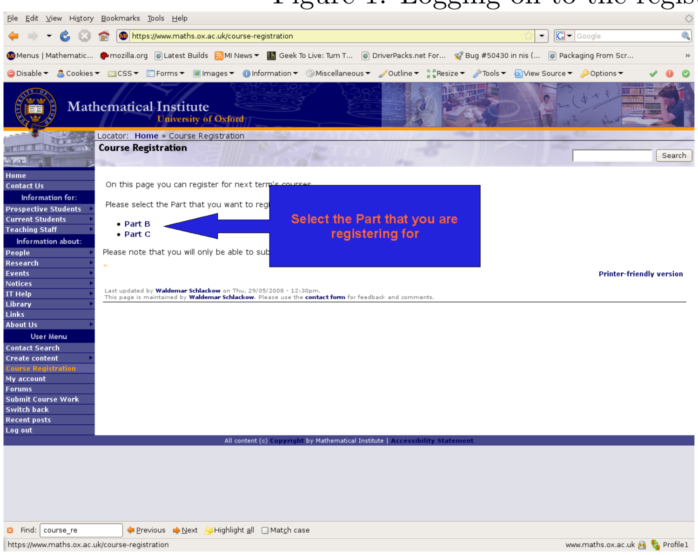

Figure 2: Select Part B.

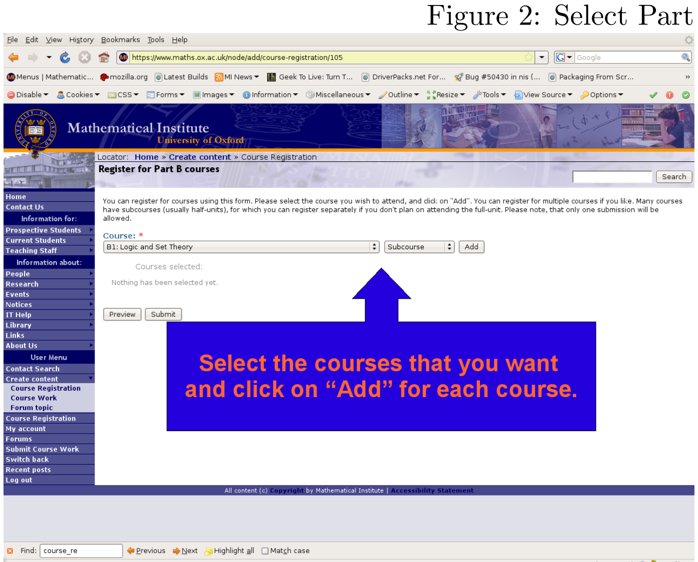

Figure 3: Select courses from the menu.

2 Schedule 1 (standard units and half-units)

2.1 B1: Logic and Set Theory

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2640), or can be taken as either a half-unit in Logic or a half-unit in Set Theory.

2.1.1 B1a: Logic — Dr Koenigsmann — 16 MT

[Option **B1a** if taken as a half-unit. OSS paper code 2A40.]

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focussing on the Completeness Theorems and the relationship between probability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, CUP, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.1.2 B1b: Set Theory — Dr Knight — 16 HT

[Option **B1b** if taken as a half-unit. OSS paper code 2B40.]

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC], $\kappa.\kappa = \kappa$ for an infinite cardinal κ.

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

2. W. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.

2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Birkhäuser, 1982). An accessible introduction to set theory.

3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.

4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.

5. G. Frege, *The Foundations of Arithmetic : A Logical-Mathematical Investigation into the Concept of Number* (Pearson Longman, 2007).

6. M. Schirn, *The Philosophy of Mathematics Today* (Clarendon, 1998). A recentish survey of the area at research level.

7. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

2.2 B2: Algebra

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2641), or available as a half-unit in B2a, or as a half-unit in B2b.

Recommended Prerequisites: All second year algebra.

2.2.1 B2a: Introduction to Representation Theory — Dr Wemyss — 16MT

[Option **B2a** if taken as a half-unit. OSS paper code 2A41.]

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

We start in a more general setting, studying modules over rings, in particular over euclidean domains, and their applications. We eventually restrict ourselves to modules over algebras (rings that carry a vector space structure). A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over $\mathbb{C}$).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem).

Learning Outcomes

Students will have a sound knowledge of the theory of non-commutative rings, ideals, associative algebras, modules over euclidean domains and applications. They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan–Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over $\mathbb{C}$ and be able to apply this.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules over euclidean domains and applications such as finitely generated abelian groups, rational canonical forms. Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem.

Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).
2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).
2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)
3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).
4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).
5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).
6. J.-P. Serre, *Linear Representations of Finite Groups,* Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.2.2 B2b: Group Theory — Dr Grabowski — 16 HT

[Option **B2b** if taken as a half-unit. OSS paper code 2B41.]

Overview

This course studies the structure of groups and their representations. As well as developing general theory, applicable to a large number of problems in group theory, we will be guided by a desire to understand finite simple groups. These groups are the building blocks from which all finite groups are made. A complete classification is beyond this course but we shall see several methods to help understand the structure of finite groups.

The methods we use will allow us to deduce key properties of a group from as little information as just the prime decomposition of its order and hence we will also exclude many possibilities for orders of simple groups. For example, we will show by solely group-theoretic methods that there are no non-Abelian simple groups of order strictly less than 60 and we will identify two infinite families of non-Abelian simple groups, namely the alternating groups of degree five or more and the projective special linear groups over finite fields.

We also introduce a key notion in the study of group representations, namely characters of a group, and study important properties of characters such as their orthogonality relations. Using character theory in a crucial way, we will show that every group having order divisible by at most two distinct primes is soluble (a theorem of Burnside) and hence there are no non-Abelian simple groups having order of this type. For a long time, this result had no proof that did not use character theory. Although such a proof is now known, this theorem and its original proof illustrate the power of representation theory in understanding the structure of groups.

Learning Outcomes

Students will be able to state and prove some of the classic theorems of finite group theory, in particular Sylow's theorems. By applying the techniques of the course, they will be able to identify whether or not there can be a simple group of a given order.

Students will also know the basic elements of the theory of group characters and will have developed a toolkit to calculate with characters.

Synopsis

We will start by recalling basic concepts of group theory and some important examples. We will mostly concentrate on finite groups but much of what we do has relevance for infinite groups too. In particular, we will recall the classification of the finitely generated Abelian groups, this being the first step towards understanding simple groups.

Next we focus on the role of the primes in group theory, where we use Cauchy's theorem on the existence of elements of prime order to help understand groups of prime power order. These groups are called p-groups and we will prove Sylow's theorems about p-subgroups of groups.

We then justify our claim that the finite simple groups are the "building blocks" of all finite groups, using composition series and the Jordan–Hölder theorem. These naturally lead to the definition and study of soluble groups and we introduce important advanced tools for studying soluble groups, such as the derived series. As well as knowing about building blocks, one also needs to know how to put these together and so we study the notion of an extension, which includes as special cases direct and semi-direct products.

Using the tools we have developed, we rule out the possibility of non-Abelian simple groups with order taking certain forms. We will show that there are actually no non-Abelian simple groups of order strictly less than 60, by purely group-theoretic reasoning. We prove that the alternating groups A_n are non-Abelian simple groups when the degree n is five or more and study another class of such groups: the projective special linear groups, which arise from matrix groups over finite fields.

We complete the course by studying group representations by means of characters, which are certain maps from the group to the complex numbers. The value on a given group element of the character associated to a representation is the trace of the representing matrix. Although this seems at first sight to discard a lot of information about the representation, in fact much remains and we will construct the character table of a group and extract a lot of structural information about the group from this. In special cases, one can even show that the character table determines the group. One surprising feature is that we can get information about finite groups from working with complex representations, i.e. over an infinite field.

Also, we will use the orthogonality relations for characters to see that even if one has only part of the character table, it is often possible to reconstruct the whole table. We will give several examples but our main application of character theory will be a proof of Burnside's $p^\alpha q^\beta$-theorem, which states that all groups whose orders have this form are soluble and hence cannot be non-Abelian simple groups.

Classification problems are a large part of what abstract algebra is about and we will gain some insight into one of the largest and most impressive such projects ever undertaken in mathematics.

Reading

1. Geoff Smith and Olga Tabachnikova, *Topics in Group Theory,* Springer Undergraduate Mathematics Series (Springer–Verlag, 2000). ISBN 1-85233-235-2

2. G.D. James and M. Liebeck, *Representations and Characters of Groups* (Second edition, Cambridge University Press, 2001). ISBN 0-521-00392-X

Further Reading

There are many books on group theory available in Oxford libraries or to buy. Any of the books below will provide an alternative perspective to the recommended text and you will find others not on this list that are equally suitable.

2. John F. Humphreys, *A Course in Group Theory* (Oxford University Press, 1996). ISBN 0-19-853459-0

3. Joseph J Rotman, *An Introduction to the Theory of Groups*, Graduate Texts in Mathematics 148 (Fourth edition, Springer-Verlag, 1995). ISBN 3-540-94285-8

4. W Ledermann, *Introduction to Group Theory* (Longman (Oliver & Boyd), 1973). ISBN 0-582-44180-3 (with A.J. Wier, Second edition. Longman, 1996. ISBN 0-582-25954-1)

5. J I Alperin and Rowen B Bell, *Groups and Representations*, Graduate Texts in Mathematics 162 (Springer–Verlag, 1995). ISBN 0-387-94526-1

2.3 B3: Geometry

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2642), or can be taken as either a half-unit in Geometry of Surfaces or a half-unit in Algebraic Curves (but see "Prerequisites").

Recommended Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Also, B3a is helpful, but not essential, for B3b.

2.3.1 B3a: Geometry of Surfaces — Dr Dancer — 16 MT

[Option **B3a** if taken as a half-unit. OSS paper code 2A42.]

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O'Neill, *Elementary Differential Geometry* (Academic Press, 1997).

2.3.2 B3b: Algebraic Curves — Prof. Joyce — 16 HT

[Option **B3b** if taken as a half-unit. OSS paper code 2B42.]

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Euler's relation. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann-Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.

2.4 B4: Analysis

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2643), or B4a may be taken as a half-unit.

Recommended Prerequisites: Part A Topology and Integration. [From Topology, only the material on metric spaces, including closures, will be used. From Integration, the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Overview

The two most important kinds of infinite-dimensional vector space are Banach spaces and Hilbert spaces; they provide the theoretical underpinnings for much of differential equations, and also for quantum theory in physics. This course provides an introduction to Banach spaces and Hilbert spaces. It combines familiar ideas from topology and linear algebra. It would be useful background for further work in analysis, differential equations, and so on.

2.4.1 B4a: Banach Spaces — Prof. Chrusciel — 16 MT

[Option **B4a** if taken as a half-unit. OSS paper code 2A43.]

Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), CHs 2, 4, 5.

2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chs 2, 4.24.3, 4.5, 7.17.4.

2.4.2 B4b: Hilbert Spaces — Prof. Batty — 16 HT

Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra. They will know the L^2-theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

Synopses

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Hilbert spaces; examples including L^2-spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

L^2-theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

Reading

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chs 3, 4.4, 6.

2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chs 3, 4.7-4.9, 4.12-4.13, 9.1-9.2.

3. N. Young, *An Introduction to Hilbert Space* (CUP, 1988), Chs 17.

Further Reading

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Ch 4.

2.5 B9: Number Theory

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2648), or B9a can be taken as half-unit (but B9b cannot).

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

2.5.1 B9a: Galois Theory — Prof. Kirwan — 16 MT

[Option **B9a** if taken as a half-unit. OSS paper code 2A48.]

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings. We also discuss general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in a discussion of some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields. The theorem of the primitive element. The existence and uniqueness of algebraic closure.

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer–Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.5.2 B9b: Algebraic Number Theory — Prof. Flynn — 16 HT

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes

11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory* (Chapman and Hall Mathematics Series, May 1987).

Further Reading

1. D. Marcus, *Number Fields* (Springer–Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.5.3 B10a: Martingales Through Measure Theory — Dr Tarrès — 16 MT

[Option **B10a** if taken as a half-unit. OSS paper code 2A49.]

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the "unknown" plays a key role, such as for instance population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, lim sup and lim inf of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, σ-algebras generated by a collection of random variables. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel-Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and L^p spaces [from Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality, orthogonal projection in L^2. The Kolmogorov Theorem and definition of conditional expectation, proof as least-squares-best predictor, elementary properties. The Radon–Nikodym Theorem [without proof, not examinable].

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition.

Uniform integrability and L^1 convergence, Levy's "Upward" and "Downward" Theorem, corollary to the Kolmogorov's Strong Law of Large Numbers, Doob's submartingale inequalities.

Examples and applications, including branching processes, and harmonic functions with boundary conditions on connected finite subsets of $\mathbb{Z}^d$.

Reading

1. D. Williams. *Probability with Martingales*, Cambridge University Press, 1995.
2. P. M. Tarres *Appendix : Notes on Fubini's theorem on* $\mathbb{R}$*, Product measures, infinite products of probability triples*, Mathematical Institute, 2009.

Further Reading

1. Z. Brzeźniak and T. Zastawniak, Basic stochastic processes. A course through exercises. Springer Undergraduate Mathematics Series. Springer-Verlag London, Ltd., 1999 [more elementary than D. Williams' book, but can provide with a complementary first reading].
2. M. Capinski and E. Kopp. *Measure, integral and probability*, Springer Undergraduate Mathematics Series. Springer-Verlag London, Ltd., second edition, 2004.
3. R. Durrett. *Probability: Theory and Examples.* (Second Edition) Duxbury Press, Wadsworth Publishing Company, 1996.
4. A. Etheridge. *A course in financial calculus*, Cambridge University Press, 2002.
5. J. Neveu. *Discrete-parameter Martingales.* North–Holland, Amsterdam, 1975.

6. S. I. Resnick. *A probability path*, Birkhaäuser, 1999.

2.6 B11a: Communication Theory — Dr Stirzaker — 16 MT

NB: B22a: Integer Programming is a very suitable complement to this course.

Level: H-level **Method of Assessment:** Written examination.

Weight: Half-unit OSS paper code 2650.

Recommended Prerequisites: Part A Probability would be helpful, but not essential. [Mathematics & Philosophy candidates should be aware that some familiarity with probability is assumed.]

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication, whilst if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

(i) Know what the various forms of entropy are, and be able to manipulate them.

(ii) Know what data compression and source coding are, and be able to do it.

(iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Error-detection and error-correction. Constraints upon the choice of good codes; the Hamming (sphere-packing) and the Gilbert–Varshamov bounds.

Reading

1. D. J. A. Welsh, *Codes and Cryptography* (OUP, 1988), Chs 1–3, 5.

2. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Ch 1–5.
3. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Ch 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).
2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

2.7 C3.1a: Topology and Groups — Dr Coward — 16MT

During 2009/10 this course will be offered at Parts B and C. However, from 2010/11 it will be offered as a Part B course only.

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code to follow.

Prerequisites

2nd year Group Theory, 2nd year Topology.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen-Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen-Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chs 1-7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Ch. 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chs. 1-7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chs. 1-5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chs. 1-4.

3 Schedule 2 (additional units and half-units)

3.1 BE "Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written extended essay.

Weight: Whole unit. OSS code 9921.

An essay on a mathematical topic may be offered as a whole unit. This would be at H-level and be 7500 words in length. It is equivalent to a 32-hour lecture course. Each project is double-blind marked. The marks are reconciled following discussion between the two assessors.

See the "Projects Guidance Notes" on the website at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/ for more information on this option and an application form.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mr Yan-Chee Yu, Room DH61, Dartingtion House, and must be received before 12 noon on Friday of Week 3 of Michaelmas Full Term. Note that a BE essay must have a substantial mathematical content. Once your title has been approved, it may only be changed by approval of Chairman of the Project's Committee.

Extended essays will be assigned USMs according to the same principles as Mathematics papers. In arriving at these marks, the relative weights (for BE Essays) given to content, mathematics, and presentation will be 25%, 50% and 25%, respectively. Here is a brief explanation of these terms and how they may be interpreted by assessors and examiners:

Content: appropriate material selected. The examiners are looking for some of your own thoughts and contributions: you must do more than rehash text books and lecture notes; you should use original sources; you must not plagiarise.

Mathematics: proofs and assertions should be correct, and the mathematics should be appropriate for the level of study. In applied topics, the derivation of the model should be properly justified.

Presentation: the examiners are looking at how clear the mathematics is and if it is well laid out; the quality of exposition (clear, well-thought out, well-organised); the English style (spelling, grammar, readability). Sources should be properly acknowledged, references should be properly cited. Give some thought to notation, choice of typeface, and numbering of equations and sections. Do not fail to number the pages. Be sure to supply complete and accurate references for all the sources used in completing the project, and be sure to cite them properly in the text.

Excellent brief advice on mathematical writing is to be found on the London Mathematical Society website http://www.lms.ac.uk/publications/documents/writing.pdf

Generally, students would have approximately 8 hours of supervision distributed over the two terms.

Submission of Extended Essay FOUR copies of your essay, identified only by your candidate number, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12 noon on Friday of week 9, Hilary Term 2010.**

3.2 O1: History of Mathematics — Dr Stedall — 16 lectures in MT and reading course of 8 seminars in HT

Level: H-level

Method of Assessment: 2 hour written examination paper for the MT lectures and 3000 word miniproject for the reading course.

Weight: Whole-unit. OSS paper code 2654 (exam), X017 (mini-project).

Recommended Prerequisites: None.

Quota: The maximum number of students that can be accepted for 2009–10 will be 20.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

  together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 500 to 3000 words);
- construction of references and bibliographies;
- verbal discussion and presentation.

Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia.*
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.

- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; sequences and series; integration; complex analysis; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 7, and 8. For each class students will be expected to prepare one piece of written work (500–1000 words) and one discussion topic.

Reading course

The Hilary Term part of the course is run as a reading course during which we will study two or three primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2010 will be decided and discussed towards the end of MT 2009. Students will be expected to write two essays (of 1500–2000 words) during the first six weeks of term. The course will then be examined by a miniproject of 3000 words to be completed during Weeks 7 to 9.

Reading

1. Jacqueline Stedall, *Mathematics Emerging: A Sourcebook 1540–1900* (OUP, 2008).
2. Victor Katz, *A history of mathematics* (brief edition, Pearson Addison Wesley, 2004), or:
3. Victor Katz, *A history of mathematics: an introduction* (third edition, Pearson Addison Wesley, 2009).

Supplementary reading

4. John Fauvel and Jeremy Gray (eds), *The History of Mathematics: A Reader* (Macmillan, 1987).

Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word miniproject at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Friday of Week 9. The miniproject will account for 50% of the marks for the course.

3.3 OCS3a Lambda Calculus & Types — Prof. Ong — 16 HT

Level: H-level. **Method of Assessment:** $1\frac{1}{2}$-hour examination.

Weight: Half-unit. OSS paper code 285.

Recommended Prerequisites: None.

Overview

As a language for describing functions, any literate computer scientist would expect to understand the vocabulary of the lambda calculus. It is folklore that various forms of the lambda calculus are the prototypical functional programming languages, but the pure theory of the lambda calculus is also extremely attractive in its own right. This course introduces the terminology and philosophy of the lambda calculus, and then covers a range of self-contained topics studying the language and some related structures. Topics covered include the equational theory, term rewriting and reduction strategies, combinatory logic, Turing completeness and type systems. As such, the course will also function as a brief introduction to many facets of theoretical computer science, illustrating each (and showing the connections with practical computer science) by its relation to the lambda calculus.

There are no prerequisites, but the course will assume familiarity with constructing mathematical proofs. Some basic knowledge of computability would be useful for one of the topics (the Models of Computation course is much more than enough), but is certainly not necessary.

Learning Outcomes

The course is an introductory overview of the foundations of computer science with particular reference to the lambda-calculus. Students will

1. understand the syntax and equational theory of the untyped lambda-calculus, and gain familiarity with manipulation of terms;
2. be exposed to a variety of inductive proofs over recursive structures;
3. learn techniques for analysing term rewriting systems, with particular reference to -reduction;
4. see the connections between lambda-calculus and computabilty, and an example of how an undecidability proof can be constructed;
5. see the connections and distinctions between lambda-calculus and combinatory logic;
6. learn about simple type systems for the lambda-calculus, and how to prove a strong normalization result;
7. understand how to deduce types for terms, and prove correctness of a principal type algorithm.

Synopsis

Chapter 0 (1 lecture)

Introductory lecture. Preparation for use of inductive definitions and proofs.

Chapters 1–3 (5 lectures)

Terms, free and bound variables, alpha-conversion, substitution, variable convention, contexts, the formal theory lambda beta, the eta rule, fixed point combinators, lambda-theories.

Reduction. Compatible closure, reflexive transitive closure, diamond and Church–Rosser properties for general notions of reduction. beta-reduction, proof of the Church–Rosser property (via parallel reduction), connection between beta-reduction and lambda beta, consistency of lambda beta. Inconsistency of equating all terms without beta-normal form.

Reduction strategies, head and leftmost reduction. Standard reductions. Proof that leftmost reduction is normalising. Statement, without proof, of Genericity Lemma, and simple applications.

Chapter 4 (2 lectures)

Church numerals, definability of total recursive functions. Second Recursion Theorem, Scott–Curry Theorem, undecideability of equality in lambda beta. Briefly, extension to partial functions.

Chapter 5 (2 lectures)

Untyped combinatory algebras. Abstraction algorithm, combinatory completeness, translations to and from untyped lambda-calculus, mismatches between combinary logic and lambda-calculus, basis. Term algebras.

Chapters 6–8 (6 lectures)

Simple type assignment a la Curry using Hindley's TA lambda system. Contexts and deductions. Subject Construction Lemma, Subject Reduction Theorem and failure of Subject Expansion. Briefly, a system with type invariance under equality.

Tait's proof of strong normalisation. Consequences: no fixed point combinators, poor definability power. Pointer to literature on PCF as the obvious extension of simple types to cover all computable functions.

Type substitutions and unification, Robinson's algorithm. Principal Type algorithm and correctness.

Reading List

Essential

1. Andrew Ker, lecture notes. Available from reception at the start of term. Comprehensive notes on the entire course, including practice questions and class exercises.

Useful Background

1. H. P. Barendregt, *The Lambda Calculus* (North–Holland, revised edition, 1984).
2. J. R. Hindley, *Basic Simple Type Theory,* Cambridge Tracts in Theoretical Computer Science 42 (CUP, 1997).
3. C. Hankin, Lambda Calculi, *A Guide for Computer Scientists,* Graduate Texts in Computer Science (OUP, 1994).
4. J. R. Hindley & J. P. Seldin, *Introduction to Combinators and Lambda-Calculus* (Cambridge University Press, 1986).
5. J.-Y. Girard, Y. Lafont, & P. Taylor, *Proofs and Types,* Cambridge Tracts in Theoretical Computer Science 7 (CUP, 1989).

3.4 N1 Undergraduate Ambassadors' Scheme — Dr Earl — mainly HT

Level: H-level

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Weight: Half-unit. OSS paper code 9919.

Recommended Prerequisites: None

Quota: There will be a quota of approximately 10 students for this course.

Co-ordinator: Dr Earl

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, N1, is a half-unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to name a second alternative half-unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

During Michaelmas term there will be a Training Day, in conjunction with the Oxford Department of Education, as preparation for working with pupils and teachers, and to provide more detail on the organisation of teaching in schools. Those on the course will also need to fill in a CRB form, or to have done so already. By the end of term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Friday of 0th week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, Richard Earl (earl@maths.ox.ac.uk).

Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (RoutledgeFalmer, 2004).

Part C (M-level) courses available in the third year

Note from the Grey Book: No candidate shall offer any unit or half unit for Part C that s/he has also offered at Part B.

3.5 C3.2: Lie Groups and Differentiable Manifolds

Level: M-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper codes to follow), or can be taken either as a half-unit in Lie Groups or a half-unit in Differentiable Manifolds.

3.5.1 C3.2a: Lie Groups — Prof. Tillmann — 16MT

[Option **C3.2a** if taken as a half-unit. OSS paper code to follow.]

Recommended Prerequisites

2nd year Group Theory, Topology, Multivariable Calculus.

Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics, and links to both algebra and analysis. Lie groups are groups which are simultaneously manifolds, so that the notion of differentiability makes sense, and the group multiplication and inverse maps are differentiable. However this course introduces the theory in a more concrete way via groups of matrices, in order to minimise the prerequisites.

Learning Outcomes

Students will have learnt the basic theory of topological matrix groups and their representations. This will include a firm understanding of root systems and their role for representations.

Synopsis

The exponential map for matrices, Ad and ad, the Campbell–Baker–Hausdorff series.

Linear Groups, their Lie algebras and the Lie correspondence. Homomorphisms and coverings of linear groups. Examples including $SU(2)$, $SO(3)$ and $SL(2;\mathbb{R}) \cong SU(1,1)$.

The compact and complex classical Lie groups. Cartan subgroups, Weyl groups, weights, roots, reflections.

Informal discussion of Lie groups as manifolds with differentiable group structures; quotients of Lie groups by closed subgroups.

Bi-invariant integration on a compact group (statement of existence and basic properties only). Representations of compact Lie groups. Tensor products of representations. Complete reducibility, Schur's lemma. Characters, orthogonality relations.

Statements of Weyl's character formula, the theorem of the highest weight and the Borel–Weil theorem, with proofs for $SU(2)$ only.

Reading

W. Rossmann, *Lie Groups: An Introduction through Linear Groups*, (Oxford, 2002), Chapters 1–3 and 6.

A. Baker, *Matrix Groups: An Introduction to Lie Group Theory*, (Springer Undergraduate Mathematics Series).

Further Reading

J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).

R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).

J. F. Price, *Lie Groups and Compact Groups* (LMS Lecture Notes 25, Cambridge, 1977).

3.5.2 C3.2b: Differentiable Manifolds — Prof. Hitchin — 16HT

[Option **C3.2b** if taken as a half-unit. OSS paper code to follow.]

Recommended Prerequisites

2nd year core algebra, topology, multivariate calculus. Useful but not essential: group theory, geometry of surfaces.

Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the B3 course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics and harmonic forms are.

Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham chomology and discussion of de Rham's theorem. Applications of de Rham theory including degree.

Riemannian metrics.

Reading

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).
2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).
3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).
4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0–3, 5–7.
5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).
6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

3.6 List of Mathematics Department units and half-units available only if special approval is granted

For details of these courses, and prerequisites for them, please consult the Supplement to the Mathematics Course Handbook, Syllabus and Synopses for Mathematics Part B 2009–2010, for examination in 2010, http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses

B5 Differential Equations and Applications	whole unit
B5a: Techniques of Applied Mathematics	half-unit

- B5b: Applied Partial Differential Equations — half-unit

B6 Theoretical Mechanics — whole unit

- B6a: Viscous Flow — half-unit
- B6b: Waves and Compressible Flow — half-unit

B7.1/C7.1: Quantum Mechanics; Quantum Theory and Quantum Computers — whole unit

- B7.1a: Quantum Mechanics — half-unit
- C7.1b: Quantum Theory and Quantum Computers — M-level — half-unit

B7.2/C7.2: Relativity — whole unit

- B7.2a: Special Relativity and Electromagnetism — half-unit
- C7.2b: General Relativity I — M-level — half-unit

B8 Topics in Applied Mathematics — whole unit

- B8a: Mathematical Ecology and Biology — half-unit
- B8b: Nonlinear Systems — half-unit

B10b: Mathematical Models of Financial Derivatives — half-unit

B12a: Applied Probability — half-unit

B21 Numerical Solution of Differential Equations — whole unit

- B21a Numerical Solution of Differential Equations I — half-unit
- B21b Numerical Solution of Differential Equations II — half-unit

B22a Integer Programming — half-unit

C5.1a Methods of Functional Analysis for Partial Differential Equations — M-level half-unit