Moderations Syllabus and Synopses 2007–2008
Honour School of Mathematics & Philosophy
for examination in 2008

Contents

1 Foreword

Syllabus

The syllabus here is that referred to in the *Examination Regulations 2007* **Special Regulations for the Honour Moderations in Mathematics and Philosophy** (p.99-101).

Synopses

The synopses give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

2 Syllabus

2.1 Pure Mathematics I

Naïve treatment of sets and mappings. Definition of a countable set. Relations, equivalence relations and partitions.

Vector spaces over $\mathbb{R}$; subspaces. The span of a (finite) set of vectors; spanning sets; examples. Finite dimensionality.

Linear dependence and linear independence. Definition of bases; reduction of a spanning set and extension of a linearly independent set to a basis; proof that all bases have the same size. Dimension of the space. Co-ordinates with respect to a basis.

Linear transformations from one (real) vector space to another. Rectangular matrices over $\mathbb{R}$. Algebra of matrices. The matrix representation of a linear transformation with respect to fixed bases; change of basis and co-ordinate systems. Composition of transformations and product of matrices.

Elementary row operations on matrices; echelon form and row-reduction. Invariance of the row space under row operations; row rank. Applications to finding bases of vector spaces.

Sums and intersections of subspaces; formula for the dimension of the sum.

The image and kernel of a linear transformation. The Rank-Nullity Theorem. Applications.

Matrix representation of a system of linear equations. Significance of image, kernel, rank and nullity for systems of linear equations. Solution by Gaussian elimination. Bases of solution space of homogeneous equations.

Invertible matrices; use of row operations to decide invertibility and to calculate inverse. Column space and column rank. Equality of row rank and column rank.

Permutations, composition of permutations. Cycles, standard cycle-product notation. The parity of a permutation; how to calculate parity from cycle structure.

Determinants of square matrices; properties of the determinant function. Computation of determinant by reduction to row echelon form. Relationship of determinant to invertibility. Determinant of a linear transformation of a vector space to itself.

Eigenvalues of linear transformations of a vector space to itself. The characteristic polynomial of a square matrix; the characteristic polynomial of a linear transformation of a vector space to itself. The linear independence of a set of eigenvectors associated with distinct eigenvalues; diagonalisability of matrices.

Division algorithm and Euclid's algorithm for integers. Polynomials with real or complex coefficients, factorization; Division algorithm and Euclid's algorithm for polynomials.

Axioms for a group. Examples, including the symmetric groups. Subgroups, intersections of subgroups. The order of a subgroup. The order of an element. Cyclic subgroups. Cosets and Lagrange's Theorem. Straightforward examples. The structure of cyclic groups; orders of elements in cyclic groups.

Axioms for a Ring. Definition of a Field and an Integral Domain.

Homomorphisms of groups. The (First) Isomorphism Theorem.

Ring homomorphisms and their kernels; First Isomorphism Theorem for commutative rings.

Euclidean Geometry in two and three dimensions approached by vectors and co-ordinates. Vector algebra. Equations of planes, lines and circles. Rotations, reflections, isometries. Parametric representation of curves, tangents, conics (normal form only) focus, directrix. Simple surfaces: spheres, right circular cones.

2.2 Pure Mathematics II

Real numbers: arithmetic, ordering, suprema, infima; real numbers as a complete ordered field. The reals numbers are uncountable.

The complex number system. The triangle inequality.

Sequences of (real or complex) numbers. Limits of sequences of numbers; the algebra of limits. Order notation.

Subsequences; every subsequence of a convergent sequence converges to the same limit. Bolzano–Weierstrass Theorem. Cauchy's convergence principle. Limit point of a subset of the line or plane.

Series of (real or complex) numbers. Convergence of series. Simple examples to include geometric progressions and power series. Alternating series test, absolute convergence, comparison test, ratio test, integral test.

Power series, radius of convergence, important examples including exponential, sine and cosine series.

Continuous functions of a single real or complex variable. The algebra of continuous functions. A continuous real valued function on a closed bounded interval is bounded, achieves its bounds and is uniformly continuous. Intermediate value theorem. Inverse Function Theorem for continuous strictly monotonic functions.

Sequences and series of functions. The uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test. Continuity of functions defined by power series.

Definition of derivative of a function of a single real variable. The algebra of differentiable functions. Rolle's Theorem. Mean Value Theorem. L'Hôpital's rule. Taylor's expansion with remainder in Lagrange's form. Binomial theorem with arbitrary index.

Step functions and their integrals. The integral of a continuous function of a closed bounded interval defined as the supremum of the integral of the step functions it dominates. Properties of the integral including linearity and the interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; integration by parts and substitution.

Term by term differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly); relationships between exponential, trigonometric functions and hyperbolic functions.

The geometry of the complex numbers. The Argand diagram. Complex numbers as a method of studying plane geometry, roots of unity, cocyclic points, Ptolemy's Theorem, equations of lines and circles (Apollonius's Theorem). Möbius transformations acting on the extended complex plane. Möbius transformations take lines and circles to lines and circles. The Riemann sphere and the conformal map to the extended complex plane. The geometry of the sphere: great circles, triangles (their six angles, angle-excess and area formula). Platonic solids.

3 Synopses of Lectures

3.1 Introduction to Pure Mathematics

3.1.1 Introduction to Pure Mathematics — Prof. Vaughan-Lee — 5 MT

There will be 5 introductory lectures in the first week of Michaelmas term.

The purpose of these introductory lectures is to establish some of the basic notation of mathematics, introduce the elements of (naïve) set theory and the nature of formal proof.

Learning Outcomes

(i) Ability to describe, manipulate, and prove things about sets and functions using standard mathematical notation.

(ii) Ability to follow and to construct proofs by Mathematical Induction (including strong induction, minimal counterexample)

(iii) (Naïve) understanding of finite, countable, uncountable sets.

Synopsis

Sets: examples including the natural numbers, the integers, the rational numbers, the real numbers; inclusion, union, intersection, power set, ordered pairs and cartesian product of sets.

The well-ordering property of the natural numbers. Induction as a method of proof, including a proof of the binomial theorem with non-negative integral coefficients.

Maps: composition, restriction, 1-1ness, ontoness, invertible maps, images and preimages.

Definition of a countable set. The countability of the rational numbers.

Reading

1. G. Smith, *Introductory Mathematics: Algebra and Analysis*, Springer-Verlag (1998), Chapters 1 and 2.
2. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis*, Third Edition (2000), Wiley, Chapter 1 and Appendices A and B.
3. C. Plumpton, E. Shipton, R. L. Perry, *Proof*, MacMillan (1984).

3.2 Pure Mathematics I

3.2.1 Linear Algebra — Dr Henke — 14 MT + Dr Curnock — 8 HT

Linear algebra pervades and is fundamental to geometry (from which it originally arose), algebra, analysis, applied mathematics, statistics - indeed most of mathematics.

Learning Outcomes

(i) To understand the general concepts of a vector space, a subspace, linear dependence and independence, spanning sets and bases.

(ii) To have an understanding of matrices and of their applications to the algorithmic solution of systems of linear equations and to their representation of linear transformations of vector spaces.

(iii) To understand the elementary properties of determinants.

(iv) To understand the elementary parts of eigenvalue theory with some of its applications.

Synopsis

Fourteen lectures in Michaelmas Term, eight lectures in Hilary Term

Fourteen lectures in Michaelmas Term

Algebra of matrices.

Vector spaces over the real numbers; subspaces. Linear dependence and linear independence. The span of a (finite) set of vectors; spanning sets. Examples.

Definition of bases; reduction of a spanning set and extension of a linearly independent set to a basis; proof that all bases have the same size. Dimension of the space. Co-ordinates with respect to a basis.

Sums and intersections of subspaces; formula for the dimension of the sum.

Linear transformations from one (real) vector space to another. The image and kernel of a linear transformation. The rank-nullity theorem. Applications.

The matrix representation of a linear transformation with respect to fixed bases; change of basis and co-ordinate systems. Composition of transformations and product of matrices.

Elementary row operations on matrices; echelon form and row-reduction. Matrix representation of a system of linear equations. Invariance of the row space under row operations; row rank.

Significance of image, kernel, rank and nullity for systems of linear equations. Solution by Gaussian elimination. Bases of solution space of homogeneous equations. Applications to finding bases of vector spaces.

Invertible matrices; use of row operations to decide invertibility and to calculate inverse.

Column space and column rank. Equality of row rank and column rank.

Eight lectures in Hilary Term

Review of a matrix of a linear transformation with respect to bases, and change of bases. [1/2 lecture]

Permutations of a finite set, composition of permutations. Cycles and standard cycle-product notation. The result that a permutation is a product of transpositions. The parity of a permutation; goodness of the definition; how to calculate parity from cycle structure.

Determinants of square matrices; properties of the determinant function; determinants and the scalar triple product.[1 1/2 lectures]

Computation of determinant by reduction to row echelon form.

Proof that a square matrix is invertible if and only if its determinant is non-zero.

Determinant of a linear transformation of a vector space to itself. [3 lectures]

Eigenvalues of linear transformations of a vector space to itself. The characteristic polynomial of a square matrix; the characteristic polynomial of a linear transformation of a vector space to itself. The linear independence of a set of eigenvectors associated with distinct eigenvalues; diagonalisability of matrices.

[3 lectures]

Reading List

1. C. W. Curtis, *Linear Algebra - An Introductory Approach*, Springer (4th edition), reprinted 1994
2. R. B. J. T. Allenby, *Linear Algebra*, Arnold, 1995
3. T. S. Blyth and E. F. Robertson, *Basic Linear Algebra*, Springer, 1998
4. D. A. Towers, *A Guide to Linear Algebra*, Macmillan, 1988
5. D. T. Finkbeiner, *Elements of Linear Algebra*, Freeman, 1972 [Out of print, but available in many libraries]
6. B. Seymour Lipschutz, Marc Lipson, *Linear Algebra* Third Edition 2001
7. R.B.J.T. Allenby *Rings, Fields and Groups* (Second Edition, Edward Arnold, 1999) [Out of print, but available in many libraries also via Amazon]

3.2.2 An Introduction to Groups, Rings, and Fields — Prof. Priestley — 8 HT and 8 TT

Abstract algebra evolved in the twentieth century out of nineteenth century discoveries in algebra, number theory and geometry. It is a highly developed example of the power of generalisation and axiomatisation in mathematics.

Aims and Objectives

The first objective is to develop some concepts of set theory as a useful language for abstract mathematics. Next we aim to introduce the student to some of the basic techniques of algebra. Thirdly, we aim to introduce group theory through axioms and examples both because groups are typical and famous examples of algebraic structures, and because of their use in the measurement of symmetry. Finally we begin the study of fields and rings which are important in the study of polynomials and their zeros.

Learning Outcomes

(i) To understand the concept of an equivalence relation;

(ii) to understand Euclidean algorithm for integers and polynomials;

(iii) to know examples and axioms of a group, field and ring, and be able to apply them in rigorous arguments.

Synopsis

HT (8 lectures)

Review of algebraic structures encountered in the course so far: $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$; $\mathbb{R}^n$ and $M_{m,n}(\mathbb{R})$.

Concept of a binary operation. Axioms for a group. Examples from above, and also set of bijections from a set to itself and invertible $n \times n$ matrices; note that a vector space is an abelian group under $+$.

Axioms for a ring (not assumed commutative with identity) Optional additions: $\cdot$ commutative; $\exists$ identity for multiplication. Examples. Definition of a field and an integral domain, with preliminary examples. Fact that a field is an ID.

[2 lectures]

The integers as an example of an integral domain which is not a field. Statement of Fundamental Theorem of Arithmetic: division algorithm and Euclidean algorithm. Polynomials with real or complex coefficients; ring structure, degree of a polynomial; Division algorithm, Euclid's algorithm; the Remainder Theorem.

[3 lectures]

Relations, equivalence relations, partitions; examples, including integers mod n.

[1 1/2 lectures]

Review of permutations of a finite set : S_n as a group. Decomposition of a permutation into a product of disjoint cycles.

[1 lecture]

Groups in general, with additional (straightforward) examples. Subgroups, with examples; intersections of subgroups.

[1 1/2 lectures]

TT (8 Lectures)

Review of HT course. More on S_n : conjugacy in S_n and relationship to normality. Cyclic groups, Order of an element. subgroup of a cyclic group is cyclic.

Homomorphisms of groups with motivating examples. Cosets and Lagrange's theorem; simple examples. Kernels, normal subgroups, The 1st Isomorphism Theorem.

[6 lectures]

Ring homomorphisms and their kernels; examples. Ideals in commutative rings, 1st Isomorphism Theorem for commutative rings; the example of integers mod n.

[2 lectures]

Remark : The number of lectures is an indicative guide.

Reading List

1. R.B.J.T. Allenby *Rings, Fields and Groups* (Second Edition, Edward Arnold, 1999) [Out of print, but available in many libraries also via Amazon]
2. I. N. Herstein, *An Introduction to Abstract Algebra.* (Third edition, Wiley 1996), ISBN - 978-0-471-36879-3.

Alternative reading

1. N.L. Biggs, *Discrete Mathematics* (OUP, revised edition).
2. Peter J. Cameron, *Introduction to Algebra*, (OUP 1998), §§1.1, 1.2, 1.3, 1.4, 3.1, 3.2, 3.3, 3.4.
3. John B. Fraleigh, *A First Course in Abstract Algebra* (Seventh edition, Pearson, 2002).
4. W. Keith Nicholson, *Introduction to Abstract Algebra* (Second edition, John Wiley, 1999), §§1.2, 1.3, 1.4, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10.
5. Joseph J. Rotman, *A First Course in Abstract Algebra* (Second edition, Prentice-Hall 2000), §§1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.3, 3.5.
6. G. C. Smith, *Introductory Mathematics: Algebra and Analysis*, Springer Undergraduate Mathematics Series 1998, Chapters 1, 2, 5.
7. Joseph Gallian *Contemporary Abstract Algebra* (Houghton Mifflin, 2006) ISBN 0618122141 [probably better as a reference].

3.2.3 Geometry I — Prof Joyce — 7 MT

These lectures give an introduction to elementary ideas in the geometry of Euclidean space through vectors.

Synopsis

Euclidean geometry in two and three dimensions approached by vectors and co-ordinates. Vector addition and scalar multiplication. The scalar product, equations of planes, lines and circles. The vector product in three dimensions. Scalar triple products and vector triple products, vector algebra. Rotations, reflections, isometries. Parametric representation of curves, tangents; conics (normal form only), focus and directrix. Simple surfaces: spheres, right circular cones.

Reading

Main text

1. J. Roe, *Elementary Geometry*, OUP (1993), Chapters 1, 2.2, 3.4, 4, 7.1, 7.2, 8.1-8.3.

Alternative reading

1. D. A. Brannan, M. F. Esplen and J. J. Gray, *Geometry*, CUP (1999), Chapter 1.
2. P. M. Cohn, Solid Geometry, *Routledge and Kegan Paul.* (1961), Chapters 1-3.
3. R. Fenn, *Geometry*, Springer (2001), Chapters 2, 5, 7.
4. J. E. Marsden and A. J. Tromka, *Vector Calculus*, McGraw-Hill, fourth edition (1996), Chapters 1, 2.
5. J. Silvester, *Geometry, Ancient and Modern*, OUP, (2001), Chapters 3, 5, 6, 7.

3.3 Pure Mathematics II

Students will already be aware of the importance and power of calculus. The aim of Analysis I, II and III is to put precision into its formulation and to make students more secure in their understanding and use. The lectures on analysis come in three parts, beginning in the first term with the properties of the real numbers and studying limits through sequences and series. The second part is concerned with continuity and differentiation, and the third part with integration. Power series and the functions they define provide a unifying theme.

3.3.1 Analysis I: Sequences and Series — Dr Earl — 14 MT

In these lectures we introduce the real and complex numbers and study their properties, particularly completeness; define and study limits of sequences, convergence of series, and power series.

Learning Outcomes

(i) An ability to work within an axiomatic framework.

(ii) A detailed understanding of how Cauchy's criterion for the convergence of real and complex sequences and series follows from the completeness axiom for $\mathbb{R}$, and the ability to explain the steps in standard mathematical notation.

(iii) Knowledge of some simple techniques for testing the convergence of sequences and series, and confidence in applying them.

(iv) Familiarity with a variety of well-known sequences and series, with a developing intuition about the behaviour of new ones.

(v) An understanding of how the elementary functions can be defined by power series, with an ability to deduce some of their easier properties.

Synopsis

Real numbers: arithmetic, ordering, suprema, infima; the real numbers as a complete ordered field. The reals are uncountable. The complex number system. The triangle inequality.

Sequences of real or complex numbers. Definition of a limit of a sequence of numbers. Limits and inequalities. The algebra of limits. Order notation: O, o.

Subsequences; a proof that every subsequence of a convergent sequence converges to the same limit; bounded monotone sequences converge. Bolzano-Weierstrass Theorem. Limit point of a set. Cauchy's convergence principle.

Series of real or complex numbers. Convergence of series. Simple examples to include geometric progressions and some power series. Absolute convergence, comparison test, ratio test, integral test. Alternating series test.

Power series, radius of convergence; important examples to include the exponential, cosine and sine series.

Reading – Main texts

1. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis*, Third Edition(2000), Wiley. Chapters 2, 3, 9.1, 9.2.
2. R. P. Burn, *Numbers and Functions, Steps into Analysis*, Cambridge University Press (2000), Chapters 2-6. (This is a book of problems and answers, a DIY course in analysis.)
3. J. M. Howie, *Real Analysis*, Springer Undergraduate Texts in Mathematics Series, ISBN 1-85233-314-6.

Alternative reading

The first five books take a slightly gentler approach to the material in the syllabus, whereas, the last two cover it in greater depth and contain some more advanced material.

1. Mary Hart, A Guide to Analysis, MacMillan (1990), Chapter 2.

2. J. C. Burkill, *A First Course In Mathematical Analysis*, CUP (1962), Chapters 1, 2 and 5.

3. K. G. Binmore, *Mathematical Analysis, A Straightforward Approach*, Cambridge University Press, Chapters, 1-6.

4. Victor Bryant, *Yet Another Introduction to Analysis*, Cambridge University Press (1990), Chapters 1 and 2.

5. G. Smith, *Introductory Mathematics: Algebra and Analysis*, Springer-Verlag (1998), Chapter 3 (introducing complex numbers).

6. Michael Spivak, *Calculus*, Benjamin (1967), Parts I, IV, and V (for a construction of the real numbers).

7. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Analysis*, Prentice Hall (2001), Chapters 1-4.

3.3.2 Analysis II: Continuity and Differentiability — Dr Qian — 16 HT

In this term's lectures we study continuity of functions of a real or complex variable variable, and differentiablity of functions of a real variable.

Learning Outcomes

At the end of the course students will be able to apply limiting properties to describe and prove continuity and differentiability conditions for real and complex functions. They will be able to prove important theorems, such as the I.V.T., Rolle's and Mean Value, and will begin the study of power series and their convergence.

Synopsis

Definition of the function limit. Examples and counter examples to illustrate when $\lim_{x \to a} f(x) = f(a)$ (and when it doesn't). Definition of continuity of functions on subsets of $\mathbb{R}$ and $\mathbb{C}$ in terms of ϵ and δ. The algebra of continuous functions; examples, including polynomials. Continuous functions on closed bounded intervals: boundedness, maxima and minima, uniform continuity. Intermediate Value Theorem. Inverse Function Theorem for continuous strictly monotone functions. Sequences and series of functions. The uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test for uniformly convergent series of functions. Continuity of functions defined by power series. Definition of the derivative of a function of a real variable. Algebra of derivatives, examples to include polynomials and inverse functions. The derivative of a function defined by a power series is given by the derived series (proof not examinable). Vanishing of the derivative at a local maximum or minimum. Rolle's Theorem. Mean Value Theorem with simple applications: constant and monotone functions. Cauchy's (generalized) Mean Value Theorem and L'Hôpital's formula. Taylor's Theorem with remainder in Lagrange's form; examples of Taylor's Theorem to include the binomial expansion with arbitrary index.

Reading – Main texts

1. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis*, Third Edition (2000), Wiley. Chapters 4-8.
2. R. P. Burn, *Numbers and Functions, Steps into Analysis*, Cambridge University Press, 2000. This is a book of problems and answers, a DIY course in analysis. Chapters 6-9, 12.
3. Walter Rudin, *Principles of Mathematical Analysis*, 3rd edition, McGraw-Hill, (1976). Chapters 4,5,7.
4. J. M. Howie, *Real Analysis*, Springer Undergraduate Texts in Mathematics Series, ISBN 1-85233-314-6.

Alternative reading

1. Mary Hart, *A Guide to Analysis*, MacMillan (1990), Chapters 4,5.
2. J. C. Burkill, *A First Course in Mathematical Analysis*, CUP (1962), Chapters 3, 4, and 6.
3. K. G. Binmore, *Mathematical Analysis*, A Straightforward Approach, CUP Chapters 7-12, 14-16.
4. Victor Bryant, *Yet Another Introduction to Analysis*, CUP (1990), Chapters 3 and 4
5. M. Spivak, *Calculus*, 3rd Edition, Publish or Perish (1994), Part III.
6. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Analysis*, Prentice Hall (2001), Chapters 5-10.

3.3.3 Analysis III: Integration — Prof Haydon — 8 TT

In these lectures we define a simple integral and study its properties; prove the Mean Value Theorem for Integrals and the Fundamental Theorem of Calculus. This gives us the tools to justify term-by-term differentiation of power series and deduce the elementary properties of the trigonometric functions.

Learning Outcomes

At the end of the course students will be familiar with the construction of an integral from fundamental principles including important theorems. They will know when it is possible to integrate or differentiate term-by-term and be able to apply this to, for example, trigonometric series.

Synopsis

Step functions, their integral, basic properties. The application of uniform continuity to approximate continuous functions above and below by step functions. The integral of a continuous function on a closed bounded interval. Elementary properties of the integral of a continuous function: positivity, linearity, subdivision of the interval.

The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; linearity of the integral, integration by parts and substitution.

The interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. Term by term integration and differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly); examples to include the derivation of the main relationships between exponential, trigonometric functions and hyperbolic functions.

Reading

1. T. Lyons *Lecture Notes*.
2. J. Roe, *Integration*, Mathematical Institute Notes (1994).
3. H. A. Priestley, *Introduction to Integration*, Oxford Science Publications (1997), Chapters 1–8. (These chapters commence with a useful summary of background 'cont and diff' and go on to cover not only the integration but also the material on power series.
4. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis*, Third Edition (2000), Wiley, Chapter 8.

3.3.4 Geometry II — Dr Earl — 8 TT

The aim of these last 8 lectures in geometry is to introduce students to some of the most elegant and fundamental results in the geometry of the Euclidean plane and the sphere.

Learning Outcomes

By the end of the course students will be able to use Möbius transforms and will be able to relate geometry and the complex plane and know the important theorems in the subject. They will know the Platonic solids and their properties and relate these to why there are precisely five.

Synopsis

The geometry of the complex numbers. The Argand diagram. Complex numbers as a methods of studying plane geometry, examples to include roots of unity, cocyclic points, Ptolemy's Theorem. Equations of lines and circles (Apollonius's Theorem), Möbius transformations acting on the extended complex plane. Möbius transformations take lines and circles to lines and circles, The Riemann sphere and the conformal map to the extended complex plane.

The geometry of the sphere: great circles, triangles (their six angles, angle-excess and area formula).

Platonic solids and the corresponding subdivision of the sphere noting Euler's formula.

Reading

1. Roger Fenn, *Geometry*, Springer (2001), Chapters 4 and 8.
2. J. Roe, *Elementary Geometry*, Oxford Science Publications (1992), Section 6.5
3. Liang-shin Hahn, *Complex Numbers and Geometry*, The Mathematical Association of America (1994).
4. George A. Jennings, *Modern Geometry and Applications*, Springer (1994), Chapter 2.
5. David A. Brannan, Matthew F. Esplen, Jeremy Gray, *Geometry*, Cambridge University Press (1999), Chapter 7.
6. Elmer G. Rees, *Notes on Geometry*, Springer-Verlag (1983), pp 23-28.