Examiners' Report: Honour Moderations in Mathematics Trinity Term 2012

October 31, 2012

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1, page 1. Overall 195 candidates were classified.

Table 1: Numbers in each class

	Numbers					Percentages %				
	2012	(2011)	(2010)	(2009)	(2008)	2012	(2011)	(2010)	(2009)	(2008)
I	60	(64)	(61)	(59)	(74)	30.77	(32.49)	(29.9)	(30.41)	(36.63)
II	123	(119)	(125)	(114)	(115)	63.08	(60.41)	(59.12)	(58.76)	(56.93)
III	4	(9)	(11)	(7)	(6)	2.05	(4.57)	(6.08)	(3.61)	(2.97)
Pass	0	(1)	(0)	(2)	(0)	0	(0.51)	(0)	(1.03)	(0)
Honours (unclassified)	1	(0)	(0)	(0)	(0)	0.51	(0)	(0)	(0)	(0)
Fail	7	(4)	(7)	(11)	(7)	3.59	(2.03)	(3.87)	(5.67)	(3.47)
Total	195	(197)	(204)	(193)	(202)	100	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for Honour Moderations in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators, however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

None.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

This is the last year under the present course structure and regulations: next year there will be a new course structure, and the examination will be Prelims, not Moderations.

D. Notice of examination conventions for candidates

The Notice to Candidates was issued at the beginning of Trinity term and contains details of the examinations and assessment. The Course Handbook contains the full Examination Conventions and all candidates are issued with this at Induction at the beginning of their first year. All notices and the Examination Conventions are on-line at
http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

- The Moderators would like to thank the academic administration team for all their work in running the examinations system for Honour Moderations: Nia Roderick (for her help throughout the year), Charlotte Turner-Smith (throughout, script-checking especially), Helen Lowe, Margaret Sloper, Sandy Patel, Vicky Archibald.
- We also thank Waldemar Schlackow and Helen Lowe for running the examination database system.
- We are very grateful to Dr C Macdonald for administration of the practical work (the MuPAD projects).
- We also thank the assessors (Drs Berczi, Davies, Lipstein, Torres) for their marking of some questions, and to tight deadlines. We are very grateful to them for their willingness to help with this unpopular task.

Timetable

The examinations began on Monday 18th June at 2.30pm and ended on Thursday 21st June at 5.30pm.

Medical certificates and other special circumstances

Following the exams there was one case passed on to the Moderators by the Proctors' Office. This case was given careful consideration in the Final Examiners' Meeting – see Section F below.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

Once the scripts had been marked and the marks entered, a team of graduate checkers under the supervision of Nia Roderick and Charlotte Turner-Smith, sorted all the scripts for each paper of the examination. They carefully cross checked against the marks scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, checking the addition. In this way a number of errors were corrected, each change signed by one of the Examiners at least one of whom was present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 195 in total. For our purposes we do not distinguish between them as they all take the same papers.

Marks for each individual examination are reported in university standardised form (USM) requiring at least 70 for a first class mark, 50-69 for a second class mark, 40-49 for a third class mark, 30-39 for a pass mark, and below 30 for a fail mark.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the percentage of candidates that might be expected in each class. This was based on the average percentage in each class over the past five years, together with recent historic data for Honour Moderations.

Moderators may recalibrate the raw marks to arrive at university standardised marks (USMs) reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have similar percentages of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates (so including Maths and Statistics candidates). A default percentage p_1 of firsts and p_2 of upper seconds in this population is entered into the database, these percentages being similar to those adopted in previous years. Note that Moderators only report candidates in the second class, rather than a divided second class, but we do carefully consider the full range of marks in deliberations.

We count down through the top p_1 percentage of candidates on a given paper which gives us the candidate at the $(100 - p_1)$-th percentile. The raw mark for the last candidate in this percentile in the ranked list is assigned a USM of 70. Let this raw mark be denoted by R_1. Continuing to count down the list of ranked candidates until another p_2 percentage of candidates is reached, the last candidate here is assigned a USM of 60. Denote this raw mark by R_2.

The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively (in this way the non-linearities are located away from the adjacent class boundaries). Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. A line segment is drawn connecting $(C_1, 72)$ to $(100, 100)$. Thus two segments of the piecewise linear graph are constructed.

Finally, the line segment through the corner at $(C_2, 57)$ is extended down towards the vertical axis as if it were to join the axis at $(0, 10)$, but is broken at the corner $(C_3, 37)$ and joined to the origin, yielding the last segment in this model. (In previous years the corner at $(C_2, 57)$ was extended as if to join $(0, 20)$, but this year we followed what we understand this year's Part B examiners did and used $(0, 10)$. We still made some adjustments afterwards, but using $(0, 10)$ gave us a starting point closer to our final judgements than $(0, 20)$ would have done.)

A first run of the outlined classification algorithm was then calculated based on the following conventions:

> First: $Av_1 \geq 70$ and $Av_2 \geq 70$
>
> Second: Not satisfying the conditions for a first class and both $Av_1 \geq 50$ and $Av_2 \geq 50$
>
> Third: Not satisfying the conditions for a second class and both $Av_1 \geq 40$ and $Av_2 \geq 40$
>
> Pass: Not satisfying the conditions for a third class and $Av_2 \geq 30$;
>
> Fail: $Av_2 < 30$

noting additionally that, in order for a candidate to pass or be awarded honours, the mark on each individual paper must be at least 30.

Here Av_2 is the average over the four written papers, and Av_1 is the weighted average over these papers together with MuPAD (with MuPAD counting as one quarter of a paper).

This gave reasonable percentages of candidates, and only a little fine tuning was then necessary.

To obtain the final classification, firstly reports from each Assessor were considered, taking into account the standard of work, comparison with previous years, and the overall level of work presented for each question on each paper. Moderators reported on their impressions of where class boundaries lay according to how the candidates had tackled the individual papers and according to the qualitative class descriptors. This gave an indication of the quality of the group for each paper. We noted carefully those candidates who were in the lowest part of each ranked list and by carefully scrutinising their scripts we were able to be clear as to who did not attain the qualitative class descriptor for a pass on the given paper. The gradients of the lower section on each paper were also considered, resulting in some slight adjustments.

Careful consideration was then given to candidates at the other class boundaries, and adjustments to the corners were considered in line with the Moderators views of the standard of candidates on each borderline.

The resulting table of the corners of the linear model is given in Table 2 on page 6. The corners are at A_1, ..., A_5, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 2: Position of corners of piecewise linear function

Paper	A_1	A_2	A_3	A_4	A_5
A	(0,0)	(27,30)	(40.8,57)	(76.8,72)	(100,100)
B	(0,0)	(29,37)	(42.6,57)	(84.6,72)	(100,100)
C	(0,0)	(32,37)	(51.3,57)	(79.8,72)	(100,100)
D	(0,0)	(30.4,37)	(53,57)	(83,72)	(100,100)

Table 3: Rank and Percentile of candidates with this or greater overall USM

Av USM	Rank	Candidates with this USM or above	%
87	1	1	0.51
86	2	3	1.54
85	4	5	2.56
83	6	6	3.08
81	7	8	4.10
80	9	11	5.64
78	12	16	8.21
77	17	18	9.23
76	19	22	11.28
75	23	25	12.82
74	26	30	15.38
73	31	34	17.44
72	35	40	20.51
71	41	48	24.62
70	49	60	30.77
69	61	65	33.33
68	66	73	37.44
67	74	81	41.54
66	82	90	46.15
65	91	103	52.82
64	104	115	58.97
63	116	124	63.59
62	125	136	69.74
61	137	142	72.82
60	143	148	75.90
59	149	159	81.54
58	160	162	83.08
57	163	165	84.62
56	166	169	86.67
55	170	173	88.72
54	174	179	91.79
53	180	180	92.31
52	181	182	93.33
51	183	183	93.85
50	184	184	94.36
48	185	185	94.87
47	186	186	95.38
46	187	187	95.90

Table 4: Continuation of the Rank and Percentage table overall USMs

Av USM	Rank	Candidates with this USM or above	%
43	188	188	96.41
42	189	189	96.92
41	190	190	97.44
40	191	192	98.46
33	193	193	98.97
32	194	194	99.49
28	195	195	100

B. Equal opportunities issues and breakdown of the results by gender

Table 5, page 8 shows the performances of candidates broken down by gender (HU = Honours unclassified).

Table 5: Breakdown of results by gender

Class	Total		Male		Female	
	Number	%	Number	%	Number	%
I	60	30.77	45	33.58	15	24.59
II	123	63.08	83	61.94	40	65.57
III	4	2.05	1	0.75	3	4.92
P	0	0	0	0	0	0
HU	1	0.51	0	0	1	1.64
F	7	3.59	5	3.73	2	3.28
Total	195	100	134	100	61	100

C. Detailed numbers on candidates' performance in each part of the exam

Performance in each individual paper is given in the tables below, Table 6, Table 7, Table 8, and Table 9, beginning on page 9.

Question Statistics for Paper A

Table 6: Statistics for Paper A

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	10.75	10.75	3.83	190	0
Q2	15.64	15.64	3.58	187	0
Q3	12.47	12.47	4.45	121	0
Q4	9.94	9.96	4.75	136	1
Q5	9.17	9.31	5.39	127	3
Q6	11.00	11.00	4.67	47	0
Q7	14.10	14.10	3.94	97	0
Q8	11.84	11.84	4.27	68	0

Question Statistics for Paper B

Table 7: Statistics for Paper B

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	12.67	12.67	4.13	162	0
Q2	5.66	5.71	3.75	69	1
Q3	8.75	8.91	4.65	87	2
Q4	13.59	13.59	5.08	182	0
Q5	16.33	16.33	2.64	194	0
Q6	12.58	12.64	6.08	138	1
Q7	13.49	13.49	4.39	109	0
Q8	11.45	11.45	5.05	31	0

Question Statistics for Paper C

Table 8: Statistics for Paper C

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	15.23	15.23	3.76	175	0
Q2	12.40	12.59	5.22	46	1
Q3	13.19	13.25	3.54	175	1
Q4	11.88	11.93	3.96	132	2
Q5	12.46	12.73	5.68	93	2
Q6	13.02	13.02	4.08	163	0
Q7	11.89	11.89	3.72	81	0
Q8	12.80	12.89	4.59	105	1

Question Statistics for Paper D

Table 9: Statistics for Paper D

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	14.66	14.91	5.53	151	3
Q2	12.52	12.52	4.38	174	0
Q3	14.51	14.51	3.36	176	0
Q4	15.60	15.60	4.20	183	0
Q5	4.8	5.05	4.10	19	1
Q6	9.68	10.14	6.85	21	1
Q7	9.51	9.72	4.89	85	2
Q8	13.15	13.17	4.61	161	1

D. Recommendations for Next Year's Examiners and Teaching Committee

In view of the change from Mods to Prelims next year, we have no specific recommendations.

E. Comments on sections and on individual questions

The following comments were submitted by the assessors.

Paper A: Pure Mathematics I

Question 1
There were some very common errors in solutions to question 1. For example in (a), the Steinitz Exchange Lemma was often wrongly or unclearly stated and in some cases even forgotten. There were no major problems with (b), but most of the candidates' answers indicated imprecise or incomplete understanding.

More candidates submitted attempted proofs for part (c) than submitted counterexamples. Most attempted proofs relied on (b)(ii).

Throughout, basic notions were often imprecisely or incorrectly stated.

Question 2
Part (a) was generally well-done. There were a few errors, one of which was (given $\{v_1, \ldots, v_k\}$ a basis for $\ker T$ extended to a basis $\{v_1, \ldots, v_n\}$ for V) to say "$w = \sum_{i=1}^{n} \alpha_i v_i \notin \ker T$, so for all $i \leq k$, $\alpha_i = 0$", which is invalid. Some candidates were not explicit enough about extending a basis for $\ker T$ to a basis for V.

In (b), (i) proved difficult (though it is similar to a part of a question on a previous paper), and some did not spot an appeal to symmetry that makes (ii) much easier. Some assumed that $\dim V_{1,1} = \dim V_{2,2}$ prematurely. There were many correct solutions to (iii).

Question 3
The most important error made by any candidate in this question was not to take on board the text in square brackets, which says, "Any properties of determinants that you use should be proved". Some candidates did this fully and got good marks. But for (a)(ii), for instance, listing the kinds of elementary row operations and their matrix representations was not adequate. Part (a)(iii) was placed where it is in the question with the proof in mind that was given in lectures. As far as I recall, every fully successful solution to (a)(iii) used that proof.

Question 4
On the whole, part (a) was done well, if at all. There was some confusion between column-rank and row-rank, understandable in view of the difficult theorem that the two are always equal.

There were few fully correct solutions to (b). This in spite of the fact that it is very easy to check whether a solution to a set of equations like this is correct. Credit was available for sentences like "I've checked these solutions and I know they're wrong but I don't have time to correct them". But even better, if an error has been found, is to go back and find it and try to fix it. The most economical method of finding an error is to substitute one's (wrong) solutions into the set of equations represented by the augmented matrix about halfway through the working. If they fit, then there must be an error earlier; if they don't, there must be an error later on. A process of repeated bisection can then be used to zero in on an error (provided the working has been laid out clearly enough).

Some candidates were insufficiently wary of dividing by quantities that might be equal to zero. This is a serious error.

Question 5
Part (a) was done well.

In (b), the most common systematic errors were: being insufficiently precise about the domains and ranges of functions (this gave rise to some incorrect counterexamples), and misunderstanding the notations $f[A]$ and $f^{-1}[A]$ where A is a set. Some attacked (iii) and (iv) believing that f^{-1} had been asserted to exist as a function, and that f was therefore invertible. Parts (v) and (vi) were generally done well.

Question 6
This question had the least number of attempts. None of them was entirely successful. Part (a)(i) and (a)(ii), the definition of the cycle-type and the criterion to be a normal subgroup respectively, did not represent a problem (for most). Yet, in a couple of cases the definition of a normal subgroup did not immediately appear to be clear to the candidate. Most attempts at solving (iii) were incomplete. The list of normal subgroups of $Sym(4)$ was either incomplete or the reasoning was unjustified: this actually happened in most cases. Part (b) was problematic for all candidates modulo some exceptions. Question (b)(ii), for example, it was not immediately clear to me that most candidates understood the geometric meaning of the element of the group G that was being asked for. Perhaps due to lack of time, finding the kernel of $Sym(4) \to Sym(3)$ was not answered correctly by most of the candidates.

Question 7
This question was arguably the one students got the most marks from, at least on average. Regarding (a), most answers were correct. However, for such a simple question, some answers were sketchy for (ii), and even for (i). Candidates that gave the sketchiest answers eventually were unable to provide a correct answer for (a)(iii) or did not even try. The answers for (b) were more homogeneous. Either they were correct (modulo some careless

parts), or they were only an attempt at playing with the algebra in hope for something to work, or the candidate did not attempt to solve it.

Part (c) proved to be the hardest block of Question 7. The problems with most of the unsuccessful attempts to solve it ranged from not using both congruences but only one to not being able to determine their solutions.

No major issues were observed in Question 7.

Question 8
There were some nice solutions handed in for Question 8. Surprisingly, showing orthogonality with respect to the standard basis for a matrix of an isometry that fixes the origin was the most troublesome part of the question. I conjecture this was due to the lack of time, since most of the candidates did present a good solution for part (a).

Paper B: Pure Mathematics II

Question 1
This was a popular question that was generally very well done. Almost all the students could give a correct statement and proof of the Bolzano-Weierstrass theorem. The substantial majority of students could also give the correct definition of a limit point of a subset of the reals, and most of those who could not do so nevertheless appeared to have the correct intuitive idea, and so were able to proceed to the remainder of the question. Most students could provide examples with the required properties. The one that posed most problems was the search for a countable subset of the reals with a countably infinite set of limit points. The final part of the question, which required the fact that a countable union of countable sets is countable, was a good way of differentiating between first and second class answers.

Question 2
This was an unpopular question, and most those who did attempt it gained fewer than 10 marks out of 20. There were, however, a couple of outstanding solutions. It seems that many students are not yet comfortable with the use of these ϵ-style arguments, which is disappointing. In the final part, which asks for a proof that $a_n = O(n) \Rightarrow x_n = O(n^2)$, some students assumed that x_n is positive for all n, which makes the solution somewhat easier. Nevertheless, partial credit was given for answers along these lines.

Question 3
This also was not a popular question, and students generally received low marks. It was surprising that the proof of part a) was incomplete in most of the solutions. Many students who attempted this part referred to the ratio test for sequences and not series, or they used ratio test for the series $\sum \frac{n^k}{a^n}$

to check that it is convergent, and then deduced that the terms of this series must tend to 0. This argument transfers the problem into another, and did not get full marks.

In part b) there were some common mistakes and inaccuracies. First of all, the majority forgot/failed to state the Comparison and/or Ratio Test for series. In (ii) when using the Comparison Test they often wrote $\sum a_n z^n \leq \sum M z^n$ without the norm $|.|$, ignoring the fact that this is a complex series, not real. In (iii) they tried to use the Comparison Test again, often assuming that $|a_n| \geq m$ for all m. And in (iii) they used the ratio test by ignoring the fact that some of the coefficients are 0, and just the minority realized that a) and b/ii can be applied here.

Question 4
This was very popular and generally done well. Almost all candidates attempting the question got close to full marks on the bookwork part a).

In b) the proof of boundedness went well by dividing $\mathbb{R}$ into two half lines and a closed interval where f is bounded and attains its bounds. However—as expected—many students did not notice that the maximum on this closed interval is not necessarily equal to the global global maximum, and therefore we need to choose our closed interval carefully.

Part c) went surprisingly wrong, just some 10 percent gave a good example, and for the rest some of the requirements on the function failed to be true: most typically they gave $h(x) = x \sin x$ which has neither a maximal nor a minimal value, but not bounded either.

Question 5
The most popular question with lots of easy bookwork and the highest average marks. All candidates attempted the question, with only 5 students getting less than 10 marks, the average is over 15 marks. However, only a few candidates had a complete solution to c(ii).

Question 6
The question was popular and generally well done. Most candidates proved Taylor's theorem in full generality and then deduced part b) as special case. In part c) many of them either forgot or failed to prove the inequality for $x < 0$: they tried to apply b) with $a = x, b = 0$ instead of using b) for $g(x) = f(-x)$.

Question 7
This was a popular question, and it was generally done very well. In the first part, many students gave proofs which did not go directly from the definition of the integral to the desired conclusion, but instead invoked much more advanced theorems, such as the fact that integration is linear. I deducted a few marks for solutions such as these. The final part was the most

challenging, and many students could not find the desired example. The simplest solution uses piecewise linear functions. Some students suggested using normal distributions with increasing variance, for which they gained full credit.

Question 8
As usual, the geometry question was not very popular. But among the submitted solutions, there were many very good answers. Most students could show that Mobius transformations preserve circlines. They could also successfully give the correct geometric picture for the later parts, which was two parallel lines, with an infinite collection of circles arranged between them.

Paper C: Applied Mathematics I

Question 1
This was a popular question. Part (a) was generally answered well although minor algebraic slips were rife. Many students failed to answer part (b) correctly, mostly because of poor skills in basic algebraic manipulation: fallacies such as $-(12 + 5x^2) = -12 + 5x^2$ were alarmingly common. There were rather more successful solutions to part (c), although many were unnecessarily longwinded.

Question 2
This question attracted relatively few attempts and even fewer successful ones. Most students managed the bookwork and derivation in parts (a) and (b) reasonably well. Although most seemed to know what was required for part (c), the straightforward differentiation and substitution involved caused problems for many. In part (d), a number of students did manage to derive the energy equation, but very few were able to manipulate it to get the required inequality.

Question 3
This was a popular question. The bookwork in parts (a) and (b) was generally remembered well, but part (c) caused great difficulties. Only a handful of students were able to integrate the differential equation $u'' = 6au^2$. Credit was given to those others who clearly demonstrated that the given solution satisfies this equation *and* the correct initial conditions.

Question 4
Most candidates remembered the general approach, and formulas, but a large proportion showed little understanding (often the correct terms were set to zero, but for the wrong reason). Although not crucial to the question, there were also very few correct sketches of the simple hyperboloid involved, and many incorrect ones.

In parts (c) and (d), many candidates solved for $dz/dt = 0$, instead of noting that, by necessity, $(dz/dt)^2 \geq 0$, and explicitly finding the corresponding range of z. Some therefore obtained the correct limits on z, but hadn't shown that the expression is actually positive between them. Another common (and related) mistake was to assert $z \geq 0$, neglecting the possibility that the particle might "fall through the hole". A surprising number of candidates failed to correctly factorise quadratic expressions which arose in parts (c) and (d), and thus obtained incorrect answers.

Question 5
Most candidates did part (a) of the question quite well, although there were many trivial mistakes in differentiation (which luckily often did not change the conclusions). Surprisingly many students failed to realise that $4x^2 - 1$ has a negative real root as well as a positive one, and therefore missed one critical point. The second derivative test for the nature of a critical point was usually applied correctly.

In part (b), most candidates correctly applied the chain rule, and obtained the differential equation satisfied by $G(u, v)$. For the second part, the crucial step was to notice that $u^2 + v^2 = e^{2x}$, after which the solution is relatively straightforward; many candidates also did this correctly.

Question 6
In general, (a) was done well. However, quite a few candidates were not comfortable with calculating the expectation of a geometric random variable in (a)(ii). There are various ways to do the different parts of (b). Part (b)(i), for example, can be done by summing a geometric series, or by one of several possible conditioning arguments, and candidates seemed to find this part at least as difficult as (b)(ii). In (b)(iii), a common error was to forget to use conditional probabilities when attempting to compute the conditional expectation.

Question 7
In (a)(i), plenty of attempts included things such as "$\mathbb{P}(X \cap Y)$" where X and Y are random variables. (Similarly, in Q6, plenty of attempts included things such as "$\mathbb{E}[X \cap A_i]$" where X is a random variable and A_i is an event.) Lots of candidates did not say "for all x and y" when defining independence for discrete random variables. Part (a)(ii) is slightly tricky: there were some elegant solutions to this, though some candidates opted to show $\mathbb{E}[f(X)g(Y)] = \mathbb{E}[f(X)]\mathbb{E}[g(Y)]$ instead, while others left a blank. In (b), plenty of candidates failed to spot that N is a binomial random variable. Those who did spot that N is binomial usually found (b) straightforward. Some attempts at (c)(iii) were able to use the probabilities $\mathbb{P}(Y_i = 1)$ to obtain $\mathbb{E}[Z]$, but most attempts missed this. There were a few nice solutions to (c)(iii), but very few got as far as writing $\mathbb{E}[Z^2]$ as $\mathbb{E}[\sum_i Y_i^2 + \sum\sum_{i,j:\, i\neq j} Y_i Y_j]$ and using the answers to (c)(i)/(c)(ii).

Question 8
The bookwork in (a) was usually done well. There were also plenty of good answers to (b)(i) and (b)(ii). However, in (b)(i), some ended up with e^v instead of $\log v$, and some missed the $1/v$ term, in the pdf of V. In (b)(ii), some attempts that were otherwise good missed that there were three cases to consider, in particular most missed that there is an $r = 0$ case.

Paper D: Applied Mathematics II

Question 1
A popular question that was attempted by most candidates. In the first part, some candidates had trouble expressing the product of two infinite sums to get an expression to which the given identities could be applied and consequently lost marks. Another popular approach to this question was to multiply $f(x)$ by its Fourier series and then apply the standard expressions (which were correctly derived in most cases) for the Fourier coefficients. Most candidates correctly found the expression for the Fourier series in part (b). Whilst there were some erroneous attempts to choose values for x or L in the last part, many candidates correctly applied the result from part (a) to the function $f(x) = x^2$ (the L-dependence cancels) to find the correct series expressions.

Question 2
This question was also very popular with candidates. Nearly all who attempted this question did well in the first half of part (a). Where marks were lost, they were often due to a failure to give a complete argument for the separation of variables. Most candidates realised that periodicity in the angular variable gave the quantisation of n.

The second half of part (a) was generally done well, although some students lost marks by simply stating that the radial solution given was correct, without actually checking or deriving it.

Most students struggled with part (b), with only a handful getting all the way to the end. Many candidates did not know where to start. A common mistake was to ignore the $n = 0$ component of the solution, derived in part (a), or to (erroneously) argue that boundedness of the solution allowed one to set the zero mode coefficients to zero.

Question 3
Most candidates attempted this question. Most candidates correctly answered part a, which asked them to derive the wave equation. Candidates had the most difficulty with part b, which asked them to solve the wave equation with a damping term (due to air resistance) and certain boundary conditions and initial conditions. The solution has oscillatory terms and an

exponentially decaying prefactor. Many students did not obtain the correct expression for the oscillation frequency and decay parameter. In part c of the question, the candidates had to show that the energy of the string was decreasing and explain the physical significance. Many candidates did not explain that the energy was being lost to the surrounding air.

Question 4
Most candidates attempted this question. Most candidates correctly answered part a, which asked them to derive the heat equation. Candidates had most difficulty with part b, which asked them to solve the heat equation with certain boundary conditions and initial conditions. Although most candidates obtained the correct general form of the solution as a Fourier series, some had difficulty evaluating the integral needed in order to compute the coefficients of the series. In part c of the question, the candidates had to show that the solution was unique. Although most students correctly pointed out that the difference between two solutions was itself a solution which initially vanishes, in some cases they did not justify why it should remain zero.

Question 5
This was not a very popular question and very few serious attempts were made. The techniques required were standard (finding the area of a curved surface given a parameterisation) but only a small number of candidates correctly found the locus $F(x, y, z) = 0$ which describes the curved surface and even fewer took the gradient of $F(x, y, z)$ to find a normal to the surface.

Question 6
This question was not as popular a some of the other questions. One the whole, those candidates with a good understanding of Gauss's flux theorem had little difficulty. Some candidates lost marks in part (a) by not stating that the gravitational field is both outward-pointing and constant on the Gaussian surface. Parts (b) and (c) were generally done well by those who attempted them.

Question 7
This question was reasonably popular. The bookwork in parts (a)(i) and (a)(ii) was recalled more-or-less correctly by most students. However, a significant minority of candidates appeared to have little grasp of the distinction between scalars and vectors, and hence were unable to manipulate correctly scalar and vector products and differential operators. Most students made some progress in part (a)(iii), but many made heavy weather of the surface integral on ∂R.

Most students made no headway at all with part (c). However, an encouraging minority produced good solutions displaying real insight.

Question 8
Many candidates did most of this question well. In (a)(i), a few candidates incorrectly included something involving $\min_i X_i$, presumably recalling the Uniform$[0, \theta]$ and similar cases, whereas (a)(i) is a simpler situation where the pdf is non-zero on $[0, \infty)$. Marks were often lost for inaccurate calculations, unclear definitions of a confidence interval, and incomplete/vague statements of the central limit theorem. Obtaining the confidence interval in (b)(ii) was generally done well, though many attempts at the corresponding part of (c) lost some marks, for example because the variance of the underlying exponential distribution was not calculated and so the resulting interval depended on an undefined quantity called σ, or because the interval obtained depended on θ itself.

F. Comments on performance of identifiable individuals

The IBM prize was awarded to the top 2 candidates.

G. Names of members of the Board of Examiners

- **Examiners:** Dr Howell, Dr Knight, Prof Lackenby, Dr Laws (Chair), Dr Reid-Edwards.
- **Assessors:** Dr Berczi, Dr Davies, Dr Lipstein, Dr Torres.