Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2012

June 25, 2013

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2012	2011	2010	2009	2008	2012	2011	2010	2009	2008
70–100	56	55	50	45	44	33.73	33.33	32.89	29.61	25.58
60–69	78	79	72	69	91	46.99	47.88	45.39	47.37	52.91
50–59	28	23	20	29	25	16.87	13.94	13.16	19.08	14.53
40–49	2	7	10	9	8	1.2	4.24	6.58	5.92	4.65
30–39	2	1	0	0	4	1.2	0.61	0	0	2.33
0–29	0	0	0	0	0	0	0	0	0	0
Total	166	165	152	152	172	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 166 take the set of four papers AC1, AC2, AO1 and AO2. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
AC1	166	64.67	11.73	66.37	10.33
AC2	166	73.35	11.55	66.62	10.05
AO1	166	62.37	14	66.34	10.85
AO2	166	75.17	15.29	68.74	12.74

B. New examining methods and procedures

None

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

A review of the structures in Part A and Mods is underway. This is likely to be implemented in 2012/13.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 20th February 2012 and the second notice on the 30th April 2012.

These can be found at http://www.maths.ox.ac.uk/notices/undergrad, and contain details of the examinations and assessments. The course Handbook contains the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

The examiners would like to express their gratitude to

- Sandy Patel and Vicky Archibald for overseeing Part A examinations during 2011/12.
- Also Waldemar Schlackow for continuing to develop the examinations database, responding to examiner requests and providing such a good framework for the examinations data.
- We would also like to thank Helen Lowe, Charlotte Turner-Smith and Nia Roderick for all their sterling work in keeping track of the scripts and marks and everything else they do during the busy examination period.
- We also thank those assessors who set their questions promptly, took care in checking and marking them, and met their deadlines. This is invaluable help for the work of the examiners.
- All the assessors and the internal examiners would like to thank the external examiner Professor Elizabeth Winstanley for her careful reading of the draft papers, scrutiny of the examination scripts and insightful comments throughout the year.

Timetable

The examinations began on Monday 18th June at 9.30am and ended on Thursday 21st June at 12.30pm.

Medical certificates and other special circumstances

See Section F

Setting and checking of papers and marks processing

As it is usual practice, questions for AC1 and AC2 were set by the examiners and also marked by them with the help of assessors. The papers AO1 and AO2 were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from the teaching committee in order to minimize the need for recalibration.
The internal examiners met in December to consider the questions for AC1 and AC2. The course lecturers were invited to comment on the notation used and in general on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to external examiner.
In a second meeting the external examiners discussed the comments of the external examiner and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term courses and at the end of Hilary and beginning of Trinity term for the Trinity term courses. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.
All examination scripts were collected by the markers from the Mathematical Institute and returned there after marking. A team of graduate checkers under the supervision of Vicky

Archibald, assisted by Charlotte Turner-Smith, sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. Here 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.
The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
AC1	75	51	32
AC2	83	61	32
AO1	76	46	26
AO2	86	58	27

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
98	1	1	0.6
93	2	3	1.81
92	4	4	2.41
91	5	6	3.61
89	7	8	4.82
86	9	9	5.42
85	10	11	6.63
83	12	12	7.23
82	13	13	7.83
81	14	16	9.54
80	17	18	10.84
79	19	19	11.45
78	20	21	12.65
77	22	23	13.86
76	24	24	14.46
75	25	27	16.27
74	28	33	19.88
73	34	39	23.49
72	40	46	27.71
71	47	51	30.72
70	52	56	33.73
69	57	61	36.75
68	62	74	44.58
67	75	86	51.81
66	87	94	56.63
65	95	100	60.24
64	101	106	63.86
63	107	117	70.48
62	118	124	74.7
61	125	130	78.31
60	131	134	80.72
59	135	138	83.13
58	139	142	85.54
57	143	146	87.95
56	147	152	91.57
55	153	157	94.58
54	158	160	96.39
52	161	162	97.59
47	163	163	98.19
42	164	164	98.8
38	165	165	99.4
35	166	166	100

B. Equal opportunities issues and breakdown of the results by gender

Table 5, page 6 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

Range	Total		Male		Female	
	Number	%	Number	%	Number	%
70–100	56	33.73	49	41.5	7	14.58
60–69	78	46.99	51	43.22	27	56.25
50–59	28	16.87	16	13.56	12	25.0
40–49	2	1.20	1	0.85	1	2.08
30–39	2	1.20	1	0.85	1	2.08
0–29	0	0	0	0	0	0
Total	166	100	118	100	48	100

C. Detailed numbers on candidates' performance in each part of the exam

Table 6 (and continued in Tables 7 and 8) give the detailed performance of candidates' on each question of the exam.

Table 6: Analysis by question

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AC1	Q1	7.30	7.30	2.17	165	0
AC1	Q2	7.13	7.13	1.95	166	0
AC1	Q3	5.54	5.54	2.44	160	0
AC1	Q4	5.04	5.04	2.18	166	0
AC1	Q5	7.93	7.93	1.47	166	0
AC1	Q6	6.49	6.49	2.36	164	0
AC1	Q7	7.48	7.48	2.07	166	0
AC1	Q8	8.53	8.53	1.92	165	0
AC1	Q9	9.60	9.60	0.97	166	0

Table 7: Analysis by question, continued

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AC2	Q1	15.96	16.55	5.79	47	5
AC2	Q2	17.74	18.46	5.78	83	6
AC2	Q3	18.35	18.80	4.83	51	4
AC2	Q4	10.74	12.67	6.45	12	7
AC2	Q5	13.34	13.55	4.63	123	7
AC2	Q6	13.79	17.53	7.21	34	18
AC2	Q7	22.61	22.61	1.93	147	0
AC2	Q8	18.31	19.51	5.23	88	11
AC2	Q9	17.80	18.38	4.10	79	10
AO1	A1	7.67	7.73	2.77	59	1
AO1	B1	6.69	6.79	3.05	53	1
AO1	C1	7.78	7.84	2.11	92	1
AO1	D1	6.30	6.36	2.99	89	4
AO1	D2	4.35	4.77	3.18	74	11
AO1	E1	5.49	5.48	2.28	101	3
AO1	E2	6.89	6.90	2.41	105	1
AO1	F1	6.33	6.43	3.03	23	1
AO1	G1	8.18	8.18	1.91	136	0
AO1	H1	7.18	7.27	2.49	44	1
AO1	J1	6.64	6.69	2.46	87	2
AO1	K1	5.68	5.62	2.72	71	3
AO1	K2	7.64	7.64	1.88	86	0
AO1	M1	7.59	7.59	2.18	95	0
AO1	M2	6.12	6.27	2.64	91	4
AO1	O1	6.99	7.04	2.19	67	3
AO1	O2	7.42	7.39	2.29	56	1
AO1	P1	9.25	9.25	1.25	77	0
AO1	P2	7.37	7.43	2.06	69	1

Table 8: Analysis by question, continued

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AO2	A2	21.81	21.8	2.69	15	1
AO2	B2	22.1	22.75	5.24	28	1
AO2	C2	20.68	21	4.40	73	5
AO2	D3	18.06	18.53	4.40	32	3
AO2	D4	17.44	17.87	5.56	23	4
AO2	E3	19.04	19.45	5.57	49	4
AO2	E4	16.73	17.0	6.96	10	1
AO2	F2	18.83	22.67	6.85	3	3
AO2	G2	14.10	15.68	6.56	62	15
AO2	H2	18.46	18.75	6.17	12	1
AO2	J2	19.56	19.88	5.56	58	4
AO2	K3	14.52	14.74	4.91	62	9
AO2	K4	18.14	18.26	4.39	50	1
AO2	M4	16.92	17.90	5.07	31	6
AO2	M5	16.79	17.79	4.83	28	5
AO2	O4	18.11	18.75	4.87	16	2
AO2	O5	17.78	19.46	6.58	35	6
AO2	P3	22.0	22.0	2.33	55	0
AO2	P4	16.48	18.3	7.12	20	3

D. Recommendations for Next Year's Examiners and Teaching Committee

None.

E. Comments on sections and on individual questions

The following comments were submitted by the assessors.

AC1

Algebra

- Question 1

  Bookwork parts were mostly fine. Most students confused in part (c) the spaces U and V and their duals U' and V'. This part of the question therefore has been marked assuming that $U = V$. Some students applied rank-nullity wrongly, others applied the hint to (imT'), reading it as $(imT)'$.

- Question 2

  Generally this question was fine, except of some miscalculations and too patchy arguments. There were some problems with working over Z_5.

- Question 3 As usual ring theory was less popular. Bookwork often was fine when the material had been learned. Most students had difficulties with part (c).

Analysis

- Question 4

  This seemed to be the hardest question in analysis. Part (a) consisted of straightforward definitions and students were comfortable with these. Part (b) (i) was answered correctly by only a small number of students: many did not attempt this part while others used inequalities involving the infima which were not thought through. In part (b) (ii) many students worringly treated infima as minima, a type of argument that was then arbitrarily repeated in part (c), ignoring the assumption of compactness. Nevertheless, in part (c), there was a significant number of students with high quality answers.

- Question 5

  The majority of students performed well, which was often the highest scoring question in analysis. Part (a) was bookwork and most students answered the questions without problems; similarly for part (b) (i). Part (b) (ii) was answered correctly by very few students. The vast majority either found the obvious solution and added a constant (ignoring part (b) (i)) or claimed that a continuously differentiable function on the complex plane is holomorphic and used part (b) (i) to deduce that there are no solutions. The latter was met very often and raises some doubts about the students' understanding of the differences between differentiability of complex and real-valued functions.

- Question 6

  The first part was straightforward and no significant problems came up. When asked to construct the appropriate conformal map, some students did not attempt this at all, whereas, for the rest and as expected, many different answers were given; mostly these were okay. However, often students neglected to check that the maps composed were conformal and, more interestingly, a minority composed the maps in the opposite order.

Differential Equations

- Question 7

  A surprising number of candidates missed the first part of the question which asked for them to simply solve the ODE. Apart from this the question separated the candidates well by questioning their knowledge of the Lipschitz continuity condition.

- Question 8

  The majority of the candidates were able to categorise the PDE successfully and from there deduce the characteristics. However, I feel that there is a misalignment of knowledge. Nearly all candidates demonstrated that the PDE was hyperbolic by showing that the auxiliary equation had a positive discriminant. They then went on to solve the auxiliary equation to show that it had two real, distinct roots. What seemed to be lacking from most candidates was the knowledge that these two pieces of information are equivalent. Thus, they could have saved themselves some time by only demonstrating the existence of solutions to the auxiliary equation. Applying the boundary conditions to the general solution, thereby allowing the equation to be completely solved, turned out to be the most difficult part.

- Question 9

  A standard Laplace transform equation which turned out to be the easiest of all the three questions. Candidates who struggled to attain any marks on the first two were able to achieve high marks on this one.

AC2

Algebra

Two very small changes to the mark scheme were made: In 2(c) I amalgamated 2+1 marks for the answer to the second sentence into 3 marks, since many students did not approach this part in the way expected. In 3(c) the first two sentences had each been allocated 2 marks which was changed to 3 marks for the first (which asked for a proof) and only 1 to the second (statement of the dimension).

- Question 1

  This question was popular with joint schools candidates, many of whom attempted two questions from the algebra section, but was much less so with mathematics candidates, the majority of whom only attempted one algebra question. Part (b) caused more difficulties than parts (a) and (c); in particular quite a number of candidates failed to write down the correct matrix, and of those who did, many were unable to calculate its characteristic polynomial.

- Question 2

  This was the most popular of the three questions by a large margin. There was a wide range of marks but plenty of good answers. However many answers were much longer than they needed to be, especially to part (b).

- Question 3

  This was the least popular question, though the number of attempts was only slightly lower than the number for Question 1. Parts (a) and (b) were mostly book work and mostly well done, though many candidates lost a mark or two by forgetting to require that irreducibles and primes should not be units or zero (divisors).

Analysis

Altogether, the students were not inclined to answer analysis questions. A large majority attempted only one of the three questions.

- Question 4

  Most of the students did not choose this question. Part (a) was a textbook question, answered correctly by most of those who attempted it. Part (b) was also very classical, and done correctly by many. Part (c) proved to be more challenging, even though it was quite close to what was done on exercise sheets. Not many students did correctly part (i), and even fewer did correctly part (ii). A common mistake was to apply Liouville's Theorem on a bounded domain.

- Question 5

  This question was chosen by almost everyone. Question (a), (b) and (c) were answered correctly by the expected amount of students. The answers to question (d) were in general less satisfactory. In (d)(i) many used the result they were asked to show within the proof, or stated that it should clearly be true. Question (d)(ii) was more challenging, and was not attempted by many.

- Question 6

  This question was chosen by a minority of students. Those who did usually did well in part (a), and reasonably well in part (b). A common mistake for part (a) was to use a contour passing through a singularity of the integrand. A handful of student noticed that this question could be successfully answered using a trigonometric substitution instead of the residue formula. The main hurdle in part (b) was to chose a convenient branch for the logarithm and the square root. If the positive real axis was excluded the computation was somewhat longer than if the negative real axis was cut out, for example. Nevertheless, a fair number of students successfully completed the computation using a logarithm defined everywhere but for non-negative reals.

Differential Equations

- Question 7

  This question was attempted by the majority of candidates and generally answered very well. Most errors appeared to be due to, for example, algebraic slips rather than a lack of understanding.

- Question 8

  Parts (a) and (b) generally answered correctly. Although most students were able to make progress with part (c), many dropped marks by not realising that the solution here is of the form "particular integral plus complementary function".

- Question 9

  Part (b vi) caused many difficulties and only relatively few scripts contained the correct domain definition. The rest of the question caused few difficulties.

AO1 and AO2

A: Introduction to Fields and B: Group Theory

There were no changes to the marking scheme. The short and long questions were very much based on the lectures and the problem sheets, testing students on main results from the course. There were minor problems only, with many students being able to give full answers.

C: Number Theory

- C1

  This question was popular and was generally well done. Many candidates lost a point as they did not (correctly) define the Euler function on 1.

- C2

  This was also a very popular question with nearly half of all candidates for Part A returning scripts. Most answers attracted alpha marks. Candidates who could not state the QR Theorem correctly were in serious difficulties.

D: Integration

There were some surprising features in the pattern of answers, most notably that only 50% of those who answered at least one question in AO1 answered both questions in AO1. Some of the candidates may covered the course only at a superficial level, hoping to pick up a few easy marks on AO1. The standard of answers on AO2 was very impressive, from a much smaller group of candidates.

- D1

  This worked much as expected. Many definitions of σ-algebra and measure were defective.

- D2

  This question was deliberately left open to allow candidates to choose any of three possible methods, but this turned out to be too optimistic. Many candidates did not see how to start, so there were a lot of low marks. Two methods had been exhibited in the lectures on a similar example, with a reference given for the third method. The great majority of the attempts which made substantial progress used differentiation through the integral, a small number tried term-by-term integration but some expanded the wrong function thereby generating a series of infinite integrals, and a tiny number used a complex change of variable without offering any justification.

- D3/D4

  The two questions produced much the same pattern of answers. There was a large proportion of high quality scripts, and very few weak ones. Only two solutions scored 25, but there were many others over 20. Marks were lost for imprecise or incomplete statements of theorems, lack of detail in applying them and failure to observe instructions to make clear statements of standard results. The questions were not easy, and the marking was quite strict.

E: Topology

- E1

  This was a popular question. It was also a successful one, because most students could attain 6/10, whereas only first class answers achieved more than this. Almost all students could give the correct definitions in the first part, and prove that Lipschitz maps are uniformly continuous. The next part, which asked the students to prove that D_A is 1-Lipschitz, was more challenging, and there were very many incorrect solutions. A common error was to get the triangle inequality the wrong way round. But there were also many very good solutions. In the final part, many students successfully realised that sequential compactness is key here. However, those solutions that just used the fact that A is closed lost marks.

- E2

  This was also a very popular question, and it was also generally very well done. Most students could give the correct definition of a basis, and provide the bookwork in the first part. It was disappointing that a substantial minority could not give the correct definition of the product topology.

- E3

  This was a popular question, with many very good solutions. It was very pleasing that the vast majority of students could deal with accumulation points well, and were able to give technically correct answers to the first parts of the question. The final example was the main way of distinguishing first class answers from the rest, and many students could not find the required set A_3.

- E4

  This question was based on the final part of the course, on quotient spaces. This is an area that many students find difficult, and so it is not surprising that this question had a relatively small number of attempts. It was generally done reasonably well, except the final part, which eluded most students.

F: Multivariable Calculus

- F1

  Almost all the candidates provided accurate definitions of continuous differentiability, manifold, tangent vector and tangent space, and stated the Chain Rule correctly (a few candidates though simply wrote the actual formula of the total derivative of the composition of two maps, without mentioning any hypotheses). The proof of the equality in (c) was given by about half of the candidates. The fact that this equality implies in (e) that $(1, 1, 1)$ is a tangent vector was noted by even fewer students.

- F2

  The candidates attempting this question were considerably fewer, and clearly very well prepared. The theoretical questions in the first part were well answered. Several students provided the statement of the Implicit Function Theorem not in the particular case of $\mathbb{R}^2$, but in full generality. Since the general theorem is much more elaborate and since it has been stated accurately each time, the students who did so quite deservedly received full marks.

  In the second part of the question applying the Implicit Function Theorem, several students found the value of the limit not by using that theorem, but by a direct calculation and the use of two Taylor series. This argument was as accurate, and since they had already applied the Implicit Function Theorem in the first part, they were awarded full marks. All the candidates that attempted the application to the Inverse Function Theorem provided answers that were quite complete.

G: Calculus of Variations

- G1

  Bookwork on Euler-Lagrange with an application derived from Mods mechanics (and done in lectures), this question attracted a lot of attempts and a high average mark.

- G2

  A question built around Euler-Lagrange with two independent variables and Lagrange multipliers leading onto a bit of Sturm-Liouville theory, this attracted quite a few attempts but ended up with rather a low average. There was quite a lot of inaccurate calculation.

H: Classical Mechanics

- H1

  A straightforward problem on Lagrangian mechanics and conserved quantities, this attracted 44 attempts, which was around the number attending the lectures, and an average over 7.

- H2

  The long problem, on moments of inertia and rigid body motion, had a lot of stages. The bookwork was close to lectures and the application was pretty straightforward, giving a highish average on a small number of attempts.

J: Quantum Theory

- J1

  This question required candidates to quantize a particle in a two-dimensional box, the three-dimensional box having been covered in lectures. Most candidates wrote down the correct Schrödinger equation inside the box, and boundary conditions, and successfully applied separation of variables to reduce to two copies of the one-dimensional box result given the hint. The majority of errors occurred in the last part of the question. In particular, a significant number of candidates assumed that the quantum number n starts at zero (despite the hint). Another common error was to assume that the two quantum numbers for the x and y directions are equal. There was also some confusion over the precise meaning of degeneracy.

- J2

  Part (a) of this question is bookwork, requiring candidates to determine the quantized energy levels of the one-dimensional harmonic oscillator. The majority of candidates gave very good answers to this part of the question. Most of the marks lost were due to careless computational errors, or for failing to be sufficiently precise. A large number of candidates also gave good answers to part (b). Common errors included writing the wrong range of integration, and not spotting the trick of integrating by parts to obtain the required formula.

K: Fluid Dynamics and Waves

- K1

  Most students were able to derive Bernoulli's equation. In part (b), few managed to calculate the pressure correctly, either because they used Bernoulli's equation (despite having just proved that the flow is not irrotational) or because they were unable to integrate the Euler equations accurately. Even fewer realised that the pressure has to be continuous, and simply bluffed the appearance of the same constant C in the two expressions for z in part (c).

- K2

  Parts (a) and (b) were generally answered well, but the correct use of Helmholtz' principle in part (c) caused many problems. Of those who did successfully find the equation of motion, most were able to demonstrate that the vortex follows the given path.

- K3

  The bookwork in this question was mostly handled well: the majority of students managed to derive Blasius' Theorem and to quote and apply the Circle Theorem. Predictably, part (c) caused great problems. Rather than factorising $\mathrm{d}w/\mathrm{d}z$ in the obvious way, most students tried (and generally failed) to expand it out and then use the quadratic formula. Most students again made life very difficult for themselves in part (d), by not simplifying the integrand before trying to use Cauchy's Residue Theorem.

- K4

  The given boundary conditions were derived more-or-less successfully, although the choice of the arbitrary functions of time in Bernoulli's Theorem confused many. Most students also managed to obtain the given dispersion relation, modulo some minor algebraic slips. Of those who realised that instability corresponds to complex values of ω, most at least got close to the required inequality. However, a worrying minority of students couldn't even relate the wavelength to the wavenumber.

M: Probability

- M1

  A straightforward Markov chain problem, with no modelling component. Many good answers.

  a) Most had adequate answers here. Many neglected to say that the sequence of states in the conditioning had to have probability 0, but if this was the only error they still received full credit.

  b) The two parts (i) and (ii) needed to be treated in a somewhat unified way. In a few answers a mistake was made in copying down the transition probabilities, or in interpreting them.

  Many students gave correct answers, but with inadequate explanation.

  Many answers only said which values of a and b were excluded by the given property (for instance, $a \neq 0$ for irreducibility and $b \neq \frac{1}{3}$ for aperiodicity), rather than formally describing the complete set of pairs of values. This was judged a reasonable interpretation of the statement of the question.

  The students had been told that aperiodicity is defined only when the chain is irreducible. No credit was deducted if they did not mention the case $a = 0$ in part (ii). The rare answers that explicitly mentioned the case $a = 0$ and said the chain is aperiodic in this case were penalised.

  (iii) Most answers here were in principle correct. The most serious error commonly made was to solve the equation $P\pi = \pi$ rather than $\pi P = \pi$.

- M2 A question about manipulating joint densities. There were no major difficulties, but surprisingly many students had difficulty recognising that the random variables are not independent. a) Most answers correct, but some confusion about what it meant to give a condition for f to be a joint density.

  biii) Some answers just gave the definition of covariance, with no explanation of how the formula related to the density.

- M3 Standard questions about computing moment generating functions, followedby advanced bookwork about asymptotic behaviour of Markov chains. No major difficulties.

- M4 This combined several standard questions about convergence, expectation and variance, and probability inequalities. a) i) and ii) were almost always correct.

  iii) A few people used Markov's inequality instead of Chebyshev's. if this was done correctly, they received half credit (2/4).

  iv) A common error here, for some reason, was to use the variance instead of the SD in the normalisation formula in the CLT.

  b) Some defined convergence in distribution as convergence of moment generating functions. Some defined it as convergence of cdfs, without the restriction to continuity points, which lost 1 point.

  c) Most did well on this. A common problem was to be confused about how to send n and ϵ to 0 at the end. A few solutions got confused earlier, putting the cdf of Y_n in the outer limits of the inequality, which doesn't really work (because the assumption is that the point is a continuity point for Y.)

- M5 Standard Poisson process questions. The opening question about the definition was correctly handled by most. The most common error was in the independence condition, where quite a few solutions only demanded independence for arrivals on two disjoint intervals, rather than arbitrarily many.

  a) and b) were done correctly by almost all. c) required recognition of the independence (and the exponential distributions) of T_1 and $T_2 - T_1$, and then a standard change of variables. A number of people attempted to use joint cdfs rather than change of variables formula on joint densities. This was very difficult to do correctly, and only one answer of this sort was reasonably correct.

  d) About half the attempts found the easy approach (using the fact that covariance is 0 for independent random variables. A few attempts to do the calculation directly from the joint distributions ended up correct; most went off track at some point.

  e) This turned out to be fairly straightforward for most people.

O: Statistics

The range of marks seemed reasonable to me on the short and the long questions. I felt during marking that Maths & Stats students performed a bit better than Maths students on the long question O5. (Perhaps there were differences, and perhaps in the opposite direction, on other questions?)

- O1

  In general, this question was done well. In (b), some candidates incorrectly gave an interval whose end-points depended on θ. In (c), plenty of attempts made minor calculation errors when finding the expected information (these were marked leniently). A few attempts obtained an "expected information" that depended on the sample $x_1, \ldots, x_n$ (this is a more serious error and lost marks).

- O2

  This question was answered very well – there was plenty to do for a full answer, and most candidates who attempted all parts did well. In the final part, some attempts made the error of saying that $\text{var}(aX)$ is the same as $a\text{var}(X)$; none of those who did this part via vectors/matrices made this error.

- O3

  In general, this question was also done well. There were some errors in (c) where the confidence interval obtained sometimes contained negative values even though the parameter σ^2 is positive. Also in (c), many confidence intervals were infinite-width (i.e. one-sided) confidence intervals, as to opposed to the finite, equal-tailed interval that I had anticipated. Many of these infinite intervals were correct answers to the question as set and received full marks.

- O4

Part (a) was straightforward and done well. In (b), many candidates resorted to long, complicated, repeated integration by parts to obtain the expectation and variance, whereas there is no need to do any integration at all. Since $f_{(r)}$ is a pdf for all r and n, and so integrates to 1, it is immediate that $\int_0^1 y^{r-1}(1-y)^{n-r}\,dy = (r-1)!(n-r)!/n!$, and this is sufficient to calculate all of the required integrals without actually doing any integration. By symmetry $\text{var}(Y_{(1)})$ and $\text{var}(Y_{(n)})$ are the same, so it is only necessary to compute one of them, but very few candidates spotted this. Most of (c) was done well though some candidates, having done (c)(i) well, then made the error of using $f(x)$ instead of $F(x)$ in (c)(ii).

- O5

  Many candidates wrote long answers for this question, but most were successful and got good marks. Part (e) was where most marks were lost: some candidates gave incorrect definitions of the p-value of the test, then proceeded to make at least one more error in order to arrive at the value given in the question. Most of the incorrect definitions of the p-value would have led to a p-value of α.

P: Numerical Analysis

- P1

  Candidates produced solutions of a high quality to this question. Almost all candidates were able to correctly define a polynomial of best least squares approximation of degree n and to confirm the statement in part (b). Part (c) was also generally well done apart from occasional algebraic mistakes.

- P2

  Candidates generally gave a good description of the first step of Richardson extrapolation but there was some confusion about how to proceed to eliminate the $\mathcal{O}(h^4)$ term of the error. Part (b) was well done. In part (c) some candiates thought that h had to be pre-computed in order to ensure that the error in the approximation was smaller than 10^{-2}. In fact, it was sufficient to use the approximations from part (b) and then confirm the error was small enough.

- P3

  Almost all candidates were able to do the bookwork in parts (a) and (b) of the question well. There were some problems in the first half of part (c) in confirming that the matrix $J(i,j,\theta)$ is orthogonal but the second half of part (c) was generally found to be straightforward. In part (d) about half of the candidates thought the matrix R would be diagonal if A is lower triangular. In fact R is still only upper triangular. There was also some confusion about how precisely to define the form of R if A is orthogonal. However, candidates generally did well on this question.

- P4

  This was not a popular question. Part (a) proved to be the trickiest bit with many small algebraic mistakes causing problems. The bookwork in part (b) was well answered and part (c) was generally well done although many candidates did not use the results of part (a) to simplify answering this part of the question.

G. Names of members of the Board of Examiners

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