Definition For topological spaces X, Y, and Z, a function $f : X \times Y \to Z$ is *continuous with respect to* x if $f|_{X \times \{y\}}$ is continuous for every $y \in Y$. Similarly, f is *continuous with respect to* y if $f|_{\{x\} \times Y}$ is continuous for every $x \in X$. The function f is *separately continuous* if f is continuous with respect to x and continuous with respect to y.

Definition A space X is *Namioka* if for every compact space Y and every metric space Z, every separately continuous function $f : X \times Y \to Z$ is continuous at each point of $D \times Y$ for some dense G_δ subset D of X

Proposition 1 (Saint-Raymond 1983) *Every completely regular Namioka space is Baire and any metrizable or separable Baire space is Namioka.*

Definition A space X is *weakly Namioka* if for every second countable space Y and every metric space Z, every separately continuous function $f : X \times Y \to Z$ is continuous at each point of $D \times Y$ for some dense G_δ subset D of X

Lemma 2 *Let Y be second countable, (M, d) be metric and $f : X \times Y \to M$ be such that f is continuous with respect to y and $f|_{X \times \{y\}}$ is continuous for every $y \in D$ for some D dense in Y. There is then a residual set A in X such that f is continuous on $A \times Y$.*

Proposition 3 *Baire spaces are weakly Namioka.*

Proposition 4 (Maslyuchenko, Mykhaylyuk, Sobchuk 1992) *Let A be a set of first category and of type F_σ in the perfectly normal space X and y_0 a non-isolated point in the completely regular second countable space Y. Then there exists a separately continuous function $f : X \times Y \to \mathbb{R}$ whose set of points of dicontinuity is $A \times \{y_0\}$.*

Corollary 5 *Perfectly normal weakly Namioka spaces are Baire.*

Theorem 6 *Completely regular, separable, weakly Namioka spaces are Baire.*

Theorem 7 (Main Theorem) *Let X be either a completely regular separable space or a perfectly normal space. Then X is Baire if and only if X is weakly Namioka.*

Definition A point $x \in X$ is a *P-point* if any G_δ set containing x is a neighborhood of x.

Definition A space X is a *P-space* if every $x \in X$ is a P-point.

Proposition 8 (Henriksen, Woods 1999) *Let X, Y be completely regular spaces, $x_0 \in X$ be a P-point and $y_0 \in Y$ have a separable neighborhood. If $f : X \times Y \to \mathbb{R}$ is separately continuous, then f is continuous at (x_0, y_0).*

Theorem 9 *Let* $x_0 \in X$ *be a P-point,* $y_0 \in Y$ *have a separable neighborhood, and* Z *be regular. If* $f : X \times Y \to Z$ *is continuous with respect to* x *and continuous with respect to* y *on a dense set* D *of* X*, then* f *is continuous at* (x_0, y_0)*.*

Example (Gruenhage, Lutzer 2000) There exists a Lindelof, hereditarily paracompact, linearly ordered P-space that is not a Baire space.

Definition A space X is *ultradisconnected* if it is crowded (has no isolated points) and if every two disjoint crowded subsets of X have disjoint closures.

Lemma 10 *First countable, ultradisconnected spaces are weakly Namioka.*

Fact Countable ultradisconnected spaces are not Baire

Example (van Douwen 1993) There exists a countable, regular, ultradisconnected space.

Example (Talagrand 1979) The function $f : [0,1] \times C_p([0,1],[0,1]) \to [0,1]$ given by $f(x,y) = y(x)$ is separately continuous and discontinuous at *every* point of $X \times Y$. It can be shown that $Y = C_p([0,1],[0,1])$, the function space with the topology of pointwise convergence, is hereditarily Lindelof and hereditarily separable.

Example (Piotrowski 1986) J.B. Brown shows there exists a separately continuous real-valued function defined on the Cartesian product of the closed interval $[0, 1]$ and the topological sum of $\mathfrak{c}$ many intervals – hence complete metric – such that the conclusion of Namioka theorem fails.

Piotrowski (2003) refines J.B Brown's techniques by constructing a separately continuous real-valued function f defined on the Cartesian product of two complete metric spaces X, Y such that the (in fact, dense G_δ) set $C(f)$ of points of (joint) continuity *fails* to contain either $A \times Y$ or $X \times B$ for *any* dense G_δ-set $A \subset X$ or *any* dense G_δ set $B \subset Y$.