Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2013

November 13, 2013

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1 and 2, page 1. Overall 178 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers 2013	Percentages % 2013
Distinction	55	30.9
Pass	103	57.87
Partial Pass	13	7.3
Fail	7	3.93
Total	178	100

Table 2: Numbers in each class (Honour Moderations)

	Numbers				Percentages %			
	(2012)	(2011)	(2010)	(2009)	(2012)	(2011)	(2010)	(2009)
I	(60)	(64)	(61)	(59)	(30.77)	(32.49)	(29.9)	(30.41)
II	(123)	(119)	(125)	(114)	(63.08)	(60.41)	(59.12)	(58.76)
III	(4)	(9)	(11)	(7)	(2.05)	(4.57)	(6.08)	(3.61)
Pass	(0)	(1)	(0)	(2)	(0)	(0.51)	(0)	(1.03)
Honours (unclassified)	(1)	(0)	(0)	(0)	(0.51)	(0)	(0)	(0)
Fail	(7)	(4)	(7)	(11)	(3.59)	(2.03)	(3.87)	(5.67)
Total	(195)	(197)	(204)	(193)	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.

- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators, however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  See Table 7 on page 10.

B. New examining methods and procedures

This is the first year of the new Preliminary Examination in Mathematics, which has replaced Honour Moderations. In summary, the main new features of the examination were:

- An increase in the number of papers from 4 to 5.
- A reduction in the length of each paper from 3 hours to $2\frac{1}{2}$ hours.
- An increase in the pass mark on each paper from a USM of 30 to a USM of 40.
- A change in the number of papers that a candidate needs to re-sit in the event of a failure. If a candidate fails one or two papers, then only these papers need to be re-taken. (This is known as a Partial Pass.) If a candidate fails three or more papers, then all papers must be re-taken. Previously, when a candidate failed any paper in Moderations, they were required to take all the re-sit papers.
- An increase in the number of re-sit papers, from 2 to 5.
- An increase in the number of Examiners, from 5 to 7.
- A change in the schedule for setting the exams. In the past, the June papers were set during the Michaelmas and Hilary terms, and the September re-sit papers were set during the Long Vacation. This year, all papers were set during the Michaelmas and Hilary terms.

Overall, the Examiners view these changes as broadly positive. We were also pleased that the examinations in this new format proceeded smoothly.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

The Examiners are broadly supportive of the new format of the first year exams. We therefore do not wish to recommend further substantial changes. However, there are several minor changes to the examination which might be made. These are discussed in more detail in Part II, Section D.

D. Notice of examination conventions for candidates

The Notice to Candidates was issued at the beginning of Trinity term and contains details of the examinations and assessment. The Course Handbook contains the full Examination Conventions and all candidates are issued with this at Induction at the beginning of their first year. All notices and the Examination Conventions are on-line at
http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

- The Moderators would like to thank the academic administration team for all their work in running the examinations system: Nia Roderick (for her help throughout the year), Charlotte Turner-Smith (throughout), Helen Lowe, Margaret Sloper, Vicky Archibald.
- We also thank Waldemar Schlackow and Helen Lowe for running the examination database system.
- We are very grateful to Dr C Macdonald for administration of the practical work (the MuPAD projects).
- We also thank the assessor (Dr Lipstein) for his marking of some questions.

Timetable

The examinations began on Monday 17th June at 2.30pm and ended on Friday 21st June at 12.00pm.

Medical certificates and other special circumstances

There was one case passed on to the Moderators from the Proctor's office. This case was given careful regard following a scrutiny of the marks.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

This year all examination scripts were collected from the Mathematical Institute rather than Examination Schools.

Once the scripts had been marked and the marks entered, a team of graduate checkers under the supervision of Nia Roderick and Margaret Sloper, sorted all the scripts for each paper of the examination. They carefully cross checked against the marks scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, checking the addition. In this way a number of errors were corrected, each change signed by one of the Examiners at least one of whom was present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 178 in total. For our purposes we do not distinguish between them as they all take the same papers.

Marks for each individual examination are reported in university standardised form (USM) requiring at least 70 for a distinction, 40-69 for a pass mark, and below 40 for a fail mark.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historic data for Honour Moderations.

Moderators may recalibrate the raw marks to arrive at university standardised marks (USMs) reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows :

Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all Mathematics candidates (including Maths and Statistics candidates). The default percentage p_1 of firsts, p_2 of upper seconds and p_3 of lower seconds and below in this population is entered into the database, these percentages being similar to those adopted in previous years. Note that Moderators only report candidates in the second class, rather than a divided second class, but we do carefully consider the full range of marks in deliberations.

We count down through the top p_1 percentage of candidates on a given paper which gives us the candidate at the $(100 - p_1)$-th percentile. The raw mark for the last candidate in this percentile in the ranked list is assigned a USM of 70. Let this raw mark be denoted by R_1. Continuing to count down the list of ranked candidates until another p_2 percentage of candidates is reached, the last candidate here is assigned a USM of 60. Denote this raw mark by R_2. Likewise R_3 is the raw mark corresponding to the percentage of p_3.

The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively. Hereby the non-linearities are located away from the adjacent class boundaries. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. A line segment is drawn connecting $(C_1, 72)$ to $(100, 100)$. Thus two segments of the piecewise linear graph are constructed.

Finally, the line segment through the corner at $(C_2, 57)$ is extended down towards the vertical axis as if it were to join the axis at $(0, 20)$, but is broken at the corner $(C_3, 37)$ and joined to the origin, yielding the last segment in this model. Here C_3 is obtained as above by extension from $(R_3, 40)$.

A first run of the outlined classification algorithm was then calculated based on the following conventions:

> Distinction: Both $Av_1 \geq 70$ and $Av_2 \geq 70$.
>
> Pass: Not meriting a Distinction and a mark of at least 40 on each paper.
>
> Fail: A mark of less than 40 on one or more papers.

Here Av_2 is the average over the five written papers, and Av_1 is the weighted average over these papers together with MuPAD (with MuPAD counting as one third of a paper).

This gave reasonable proportions of candidates, and only a little fine tuning was then necessary.

To obtain the final classification, firstly reports from each Assessor were considered, taking into account the standard of work, comparison with previous years, and the overall level of work presented for each question on each paper. Moderators reported on their impressions of where class boundaries lay according to how the candidates had tackled the individual papers and according to the qualitative class descriptors. This gave an indication of the quality of the group for each paper. We noted carefully those candidates who were in the lowest part of each ranked list and by carefully scrutinising their scripts we were able to be clear as to who did not attain the qualitative class descriptor for a pass on the given paper. The gradients of the lower section on each paper were also considered resulting in some slight adjustments.

Careful consideration was then given to candidates at the other class boundaries, and adjustments to the corners were made in line with the Moderators views of the standard of candidates on each borderline.

The resulting table of the *corners* of the linear model is given in Table 3 on page 7. The corners are at A_1, ..., A_5, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 3: Position of corners of piecewise linear function

Paper	A_1	A_2	A_3	A_4	A_5
I	(0,0)	(34,40)	(49,57)	(79,72)	(100,100)
II	(0,0)	(31,40)	(40,57)	(70.4,72)	(100,100)
III	(0,0)	(51.6,57)	(78.6,72)	(100,100)	
IV	(0,0)	(32,42)	(41.3,57)	(69.8,72)	(100,100)
V	(0,0)	(30,39)	(41.8,57)	(77.8,72)	(100,100)

Table 4: Rank and Percentile of candidates with this or greater overall USM

Av USM	Rank	Candidates with this USM or above	%
91	1	2	1.12
90	3	3	1.69
88	4	4	2.25
87	5	5	2.81
85	6	6	3.37
84	7	8	4.49
83	9	10	5.62
82	11	12	6.74
81	13	14	7.87
80	15	16	8.99
79	17	18	10.11
77	19	23	12.92
76	24	26	14.61
75	27	30	16.85
74	31	34	19.10
73	35	36	20.22
72	37	40	22.47
71	41	47	26.40
70	48	55	30.90
69	56	61	34.27
68	62	67	37.64
67	68	75	42.13
66	76	86	48.31
65	87	100	56.18
64	101	108	60.67
63	109	113	63.48
62	114	120	67.42
61	121	131	73.60
60	132	134	75.28
59	135	139	78.09
58	140	145	81.46
57	146	153	85.96
56	154	156	87.64
54	157	158	88.76
53	159	163	91.57
51	164	165	92.70
50	166	166	93.26

Table 5: Continuation of the Rank and Percentage table overall USMs

Av USM	Rank	Candidates with this USM or above	%
49	167	167	93.82
47	168	169	94.94
45	170	170	95.51
44	171	172	96.63
43	173	175	98.31
42	176	176	98.88
41	177	177	99.44
39	178	178	100

B. Equal opportunities issues and breakdown of the results by gender

Table 6, page 9 shows the performances of candidates broken down by gender.

Table 6: Breakdown of results by gender

Class	Total		Male		Female	
	Number	%	Number	%	Number	%
Distinction	55	30.9	41	33.06	14	25.93
Pass	103	57.87	71	57.26	32	59.26
Partial Pass	13	7.3	6	4.84	7	12.96
Fail	7	3.93	6	4.84	1	1.85
Total	178	100	124	100	54	100

C. Detailed numbers on candidates' performance in each part of the exam

Performance in each individual paper is given in the tables below, Table 7, Table 8, Table 9, Table 10, and Table 11 beginning on page 10. Table 12 gives, for each paper, the number of candidates who received a USM of less than 40 on that paper.

Question Statistics for Mathematics I

Table 7: Statistics for Mathematics I

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	12.75	12.75	4.70	171	0
Q2	9.29	9.29	4.60	125	0
Q3	12.78	12.84	3.73	129	1
Q4	12.45	12.45	4.87	105	0
Q5	12.97	13	5.24	60	1
Q6	13.7	13.7	3.71	140	0
Q7	17.1	17.1	3.75	154	0

Question Statistics for Mathematics II

Table 8: Statistics for Mathematics II

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	12.38	12.38	2.92	176	0
Q2	13.26	13.26	3.9	176	0
Q3	13	13	8.49	2	0
Q4	8.18	8.18	4.84	94	0
Q5	15.03	15.03	4.4	115	0
Q6	11.13	11.13	4.08	145	0
Q7	8.1	8.1	5.32	171	0

Question Statistics for Mathematics III

Table 9: Statistics for Mathematics III

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	15.33	15.33	3.82	168	0
Q2	17.18	17.18	3.62	104	0
Q3	12.82	13.05	4.48	82	2
Q4	11.46	11.51	2.6	131	1
Q5	14.96	14.96	2.53	95	0
Q6	13.14	13.14	4.13	128	0
Q7	10.22	10.22	5.16	177	0

Question Statistics for Mathematics IV

Table 10: Statistics for Mathematics IV

Question Number	Average Mark		Std Dev	Number of Attempts	
	All	Used		Used	Unused
Q1	7.64	7.64	5.63	80	0
Q2	13.18	13.18	4.02	177	0
Q3	7.14	7.14	4.31	161	0
Q4	12.54	12.63	5.22	110	1
Q5	14.99	14.99	3.98	158	0
Q6	12.75	12.75	4.24	163	0
Q7	11	11.21	4.88	34	2

Question Statistics for Mathematics V

Table 11: Statistics for Mathematics V

Question	Average Mark		Std	Number of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	13.35	13.35	5.03	146	0
Q2	15.21	15.21	4.41	156	0
Q3	12.34	12.7	6.16	54	2
Q4	13.85	13.85	4.49	164	0
Q5	11.36	11.36	5.57	149	0
Q6	10.12	10.12	5.7	42	0
Q7	9.22	9.22	4.79	174	0

Failure rates on individual papers

Table 12: The number of failures for each paper

Paper	Number	%
I	6	3.4
II	8	4.5
III	2	1.1
IV	7	3.9
V	13	7.3
Mupad	2	1.1

D. Recommendations for Next Year's Examiners and Teaching Committee

Workload. The introduction of Prelims has, without doubt, increased the total workload on the Faculty, simply because 10 papers now need to be set, rather than 7. However, with the increase in the number of Examiners, the amount of work required of each individual Examiner in setting the papers was only slightly greater than in previous years. The Examiners were able to reduce their reliance on Assessors during the marking process. Our recommendation is that the number of Prelims Examiners remains unchanged in future years.

Schedule for setting the papers. This year, both the June and September papers were set during the Michaelmas and Hilary terms. We recommend that a similar schedule is adopted in future years. In the past, the re-sit examinations were set during the Long Vacation, but with the significant increase in the number of re-sit papers, this is no longer feasible.

Failure rate. The number of candidates required to re-sit every paper has remained unchanged from previous years. However, there has been a significant increase in the number of candidates who have been required to re-sit at least one paper. The Examiners believe that this is an accurate reflection of the abilities of the weaker candidates, and trust that it will be beneficial for these students to devote more time to the topics that they have struggled with, so that they are sufficiently well-prepared to go into the second year.

The length of questions. With the length of each paper reduced from 3 hours to $2\frac{1}{2}$ hours, the average length of time that each candidate may spend on a question has been reduced from 36 minutes to 30 minutes. The Examiners were very much aware of this, and set the questions accordingly. As a result, we do not think that the candidates were placed under excessive time pressure. However, we view it as important that, in future years, the length of each question does not increase.

Mupad. Accompanying the change to Prelims, there was a modification in the way that Mupad was dealt with in the examination. It is now a requirement that candidates pass Mupad overall, and if they do not, they will be required to re-take the projects. In the Examiners' view, it is critically important that, before a candidate is failed on this basis, the Examiners should have a chance to review their project and be satisfied that it does merit a fail mark. In addition, since the pass mark is set at a USM of 40, some rescaling of Mupad marks may be required. This year, we had to contact the Mupad co-ordinator Dr Macdonald, who had marked the projects, during the final Examiners meeting. We envisage that, in future, a similar discussion between the Mupad co-ordinator and the Examiners is likely to be required. We therefore *recommend* that, in future years, the Mupad co-ordinator is present during the part of the Examiners meeting that deals with Mupad.

Comments on the syllabus. The change to Prelims coincided with a substantial overhaul to the first year syllabus. We would like to pass on to the Teaching Committee some observations of the effect of this syllabus change on the students' examination performance:

Paper I: Anecdotally, many students found some aspects of the new Linear Algebra II course challenging. However, the examination questions were carefully set, so that they were based solely on the material in the syllabus. As a result, the candidates fared reasonably well on these questions. We recommend no changes here.

Paper II: Analysis is an area that weaker students have always found difficult. The level of the paper was about right this year, in the Examiners' view. We recommend no changes next year.

Paper III: No particular issues arose this year with this part of the syllabus.

Paper IV: In the Examiners' view, many of the weaker students found the Dynamics course challenging this year. Some relatively advanced new material has been incorporated into the same number of lectures. However, the questions in the exam were carefully calibrated, with the result that the majority of candidates were able to make reasonable attempts at them.

Paper V: Surprisingly, this was the paper with the largest number of failures. One interpretation of this is that vector calculus can be very challenging at the first encounter. In the old first year, this material was tackled very gently, first in 2D before generalising to 3D. In the new structure, the approach is much more direct, and this appears to have caused difficulties for the weaker students. It is the Examiners' view that the questions on Fourier Series and PDEs were of a very similar standard to the problem sheets and at the correct level of difficulty. Overall, it is the Examiners' view that, although the questions on Paper V were of a comparable level to previous years, the courses seemed to cover more material and weaker students struggled to cover these newly expanded courses sufficiently to cope with the exam.

Compulsory questions. The Faculty may wish to consider further the precise way that the examination papers are divided into parts. In particular, it is currently anomalous that Statistics is the only Hilary term course with its own part. As a consequence, the only compulsory question on Hilary term material is in Statistics.

The Examiners discussed this matter at some length. It is our view that compulsory questions have some disadvantages. They permit only one topic, in what may be a diverse course, to be examined. They also place pressure on the setter of the question to get the level right. On the other hand, by setting compulsory questions on Trinity term material, they do encourage students to devote sufficient time to the courses taking place this term.

There was no consensus among the Examiners on whether the current arrangements for Statistics are problematic. Moreover, if this anomaly is to be removed, there were differing suggestions for achieving this. One option is to amalgamate the Probability and Statistics questions, thereby forming a single part of Paper III. Students would then be required to answer three out of four of these questions. Another option is to divide other papers into more parts, for example by dividing the Geometry and Dynamics questions into two parts in Paper IV. We leave it to the Teaching Committee to discuss this matter further.

E. Comments on sections and on individual questions

Paper: Mathematics I

Question 1
Most candidates attempted this. Part (a) was generally done well. In part (b), many students stated the Steinitz Exchange Lemma, but altering either the hypothesis or the conclusion or both. Often it was not clear how the lemma was used. The steps of the proofs were not always justified clearly and not enough details were given.

Many candidates did not prove that bases have the same cardinality and it is possible that some of them simply forgot this part of the question.

Part (c) was more challenging. In their answer to (c)(ii), many candidates incorrectly said that a basis of the intersection of two subspaces is the intersection of the bases.

Question 2
A fair number of candidates attempted this and many found the question challenging.

Part (a) was coursework but many students failed to prove that the basis that they produced was indeed linearly independent.

In part (b) several mistakes were made. Some candidates proved uniqueness but did not show existence. Some defined the map but did not show that it is linear. Most candidates who did this correctly used matrices, which was unnecessary but correct. Few defined the linear map directly.

In part (c) many candidates did the second part producing a counterexample. Few students correctly answered the first part of the question. This required one to imitate the proof of part (a), choosing a basis appropriately.

Question 3
Many candidates attempted this.

In part (a) the definitions were generally correct, except that some candidates missed that in one of the operations the scalar is non-zero. The proof was done well even though most candidates used elementary matrices instead of giving a more direct proof.

Part (b): Few candidates managed to solve the system completely. Several candidates forgot to treat all cases so assumed arbitrarily that the parameter did not take the values 0,1. Many candidates did not swap rows, thereby making the calculations unnecessarily complicated. Marks were given for following a correct general method even if numerical mistakes were made.

Question 4
A fair number of candidates attempted this.

Most students did well with the first part.

In part (b), quite a few candidates miscalculated the characteristic polynomial. Several of them claimed subsequently that some eigenvalue has geometric multiplicity equal to 0 which is absurd. Apart from that, students did not have difficulties determining the eigenvectors. Very few managed to do the last part finding the matrix P as most candidates did not apply the Gram-Schmidt procedure to find a set of orthonormal vectors. A common mistake was to just divide by the norm and claim the the set of vectors obtained is orthonormal.

Question 5
A somewhat unpopular question, though reasonably well attempted, although some scripts did not address the if-and-only-if nature of the result in (a). The start of part (b) was well done, but few progressed to explain why there are 18 permutations in the centralizer of (123)(456). Finding two non-commuting elements, e.g. (123) and (14)(25)(36), was sufficient to show this subgroup is non-abelian.

Question 6
A popular question, reasonably well done. When asked to show that kernels are normal subgroups, quite a few scripts missed easy marks and failed to show that kernels are indeed subgroups, showing only that they are closed under conjugacy. Amongst weaker scripts there were quite a few muddled versions of the Isomorphism Theorem. Also, quite a few candidates laboured to determine the normal subgroups of S_3, when it would have been sufficient to simply note the conjugacy classes and to apply Lagrange's Theorem. For these normal subgroups, many could identify the corresponding quotient group but could not apply this information appropriately. Some students, usually unsuccessfully, aimed to determine facts about the images of generators of S_3.

Question 7
A very popular question and very well done. Many students proved comfortable with the group actions bookwork and showed sound knowledge of the conjugacy classes of D_8 (though the latter wasn't explicitly needed to answer the question). Some weaker solutions tried to work backwards from the answer, concluding incorrectly that one of the conjugacy classes was size 3.

Paper: Mathematics II

Question 1
Many candidates attempted this.

Part (a) was generally done well but some candidates did not manage to do (a)(iii) often quoting what they had to prove.

Part (b) was more challenging but many candidates attempted several parts of this. Quite a few managed to do (b)(iii) realizing that it follows from the definition of the limit and several managed to do parts (i) and (ii). Most used the hint for part (i) but some gave alternative direct proofs. Several candidates made elementary mistakes when manipulating inequalities. Few candidates did the more challenging parts (iv) and (v). In part (v) many candidates tried to use the algebra of limits but gave incorrect arguments.

Question 2
This question was generally answered very well. Almost all the candidates could provide a proof that Cauchy sequences of real numbers converge. The second part of the question, which required the students to examine a specific non-Cauchy sequence, was less well done. Many found it difficult to provide precise proofs. Although considerable partial credit was given to many attempts, answers such as

$$\sin(\pi\sqrt{n+1}) \to \sin(\pi\sqrt{n}) \Rightarrow a_{n+1} - a_n \to 0$$

received no marks.

Question 3
This was an extremely unpopular question, with only two attempts! The most likely explanation for this is the fact that the bookwork was less familiar than in the other questions. Another possible conclusion that one might draw is that students are less comfortable with power series than they should be (see Question 4 below). However, it was a largely straightforward question, but perhaps a little long.

Question 4
There was some very good answers to this question, but there were also quite a few weaker ones. It was surprising that, although almost everyone could give the correct definition of a uniformly convergent sequence of functions, many people had little idea what this meant in practice. Many students erroneously claimed that if a sequence of functions f_n satisfies $f_n(1/n) = 1$ for all n, then the sequence can converge uniformly to the zero function.

The proof that $f_n(x_n) \to f(x)$ was typically attempted quite well. The simplest correct approach is to compare $f_n(x_n)$ with $f(x_n)$ (using uniform convergence), and $f(x_n)$ with $f(x)$ (using the continuity of f). Some solutions tried to compare $f_n(x_n)$ with $f_n(x)$, and then $f_n(x)$ with $f(x)$. This method does not work, but some partial credit was given.

Few students could handle the final part of the question, which was actually quite easy. Perhaps this also can be explained by their relative unfamiliarity with power series.

Question 5
This was a very popular question that was generally answered very well. Many students were able to recall the bookwork, and then to apply it well to find the derivative of an inverse function. Some students attempted to produce a proof of the chain rule that was not in the lecture notes. These proofs were typically largely correct, but many omitted to consider the possibility that $f(x)$ might equal $f(x_0)$ when $x \neq x_0$, and so lost some marks there.

Question 6
Almost all the candidates who attempted this question could recall Cauchy's form of the mean value theorem and to use it to prove L'Hôpital's theorem. However, very few fared well with the later part of the question. The hint suggested that the students considered setting $f(x)$ to be $x^m \sin(x^n)$, (when $x \neq 0$) for integers m and n. Only when n is negative does this lead easily to the required examples. Unfortunately, the majority of the students struggled with positive values of n which often led to long but erroneous solutions.

Question 7
A mandatory question, which tended to provide very good solutions (in a minority of cases) or poor ones barely addressing the bookwork. Many scripts did not respond to the first part as asked, instead showing that f is Riemann integrable; had this ever been in any doubt then it would have been sufficient to just note it is continuous, being the uniform limit of continuous functions. For the first part of (c), it was sufficient simply to note that exponentials dominate polynomials, to prove this using the exponential power series, to apply L'Hopital's Theorem, or to quote $n^k/a^n \to 0$ when $a > 1$. Many scripts simply then gave up after this. Those continuing to the final parts of the question often did well, though a minority incorrectly desired to apply the pointwise convergence of the integrand on $(0, 1]$ to results about the integral.

Paper: Mathematics III

Question 1
This question was quite popular and done reasonably well. Both parts of part (a) could be done with integrating factors, although some candidates

also correctly solved the first problem by finding a complementary function and particular integral. In part (b) all candidates found the correct eigenvalues, many found the correct eigenvectors and most knew how to construct the general solution from this. Very few candidates took advantage of the fact that, once the eigenvalues are found, an ansatz for $x(t)$ and $y(t)$ can easily be constructed and refined by submitting into the coupled differential equations. Most candidates who attempted part (c) found the correct complementary function, correctly dealing with the repeated root. A surprisingly small number managed to suggest a reasonable ansatz for the particular integral, leading to many confused attempts.

Question 2
About half of the candidates attempted this question and most did well. In part (a), some candidates made calculational errors in using the chain rule to express the differential equation in terms of ξ, η. In part (b), a few candidates did not solve the differential equation in terms of ξ, η and then express the solution in terms of x, y. Rather, they took the more complicated approach of plugging the solution in terms of x, y directly into the differential equation and attempting to verify that it is indeed the solution.

Question 3
This question was relatively unpopular but generally answered well by those who attempted it. In part (a), most managed to locate and classify the critical points but not to show that these were the only ones. In part (b), most applied the method of Lagrange multipliers correctly but the straightforward manipulation of the resulting algebraic equations caused many problems.

Question 4
Parts (a) and (b) were were done well, though some candidates were not able to use the binomial theorem to find the pgf of X as $G_X(s) = (1 - p + ps)^n$. Part (c) was found much harder. The third sentence of (c)(i) gives some information about the first ball withdrawn: it is red. This means the probability that the remaining ball is red is a conditional probability – the conditional information being that the first ball was red. However most candidates calculated an unconditional probability in (c)(i) (e.g. calculated the probability that the second ball is red when knowing nothing about the result of the first draw). The same is true in (c)(ii): we know the result of the first draw, hence the probability to be calculated is again a conditional probability.

Question 5
Most candidates were comfortable manipulating random variables in (a), (b) and (c)(i)/(ii). Many were also able to show that X and Y are not independent, but only a few were able to complete the last part (the first bit of (c)(iii) gives a hint about how to do (c)(iv)).

Question 6

Part (a) was done well, though in (a)(i) some candidates did not check that their pdf was nonnegative and some did not calculate the expectation. In (b)(i) some candidates found the correct transformation which mapped random variable V on $[a, b]$ to a random variable on $[0, 1]$ but did not justify that their transformed random variable was uniform on $[0, 1]$.

Question 7

It seems as though quite a lot of candidates ran out of time on this paper – quite a few indicated that they had run out of time while doing this question, so the mark distribution on this question will surely be affected by this. Part (a) was generally done well, though some candidates did not check whether or not their estimators were unbiased. Most candidates were able to obtain the likelihood in (a)(iii) where m random variables have mean θ and $n-m$ have mean 2θ, though there were some algebraic errors thereafter. In part (b) sketching the likelihood is a good idea: a (correct) sketch makes it obvious where the likelihood is maximised. Some of those who did not draw the likelihood did not give a clear explanation of where their maximum likelihood estimator came from.

Paper: Mathematics IV

Question 1

This question was not very popular. The majority made some progress with the first result of part (a), though often by inelegant longwinded methods. A sizeable minority displayed no grasp at all of the distinction between vectors and scalars. In part (b), most could sketch the surfaces but were unable to find the corresponding equations and therefore could not find their intersection.

Question 2

This question was popular and relatively well answered. The bookwork in part (a) was generally handled well. In part (b), a common error was to try and solve for $u(\theta)$ before evaluating the angular velocity h. Only the strongest students knew how to reintroduce t to answer part (c).

Question 3

This question was popular but the attempted solutions were generally of very poor quality. There were many garbled derivations and uses of the conservation principles needed in part (a). In part (b), many students were unable to manipulate the energy inequality accurately and instead simply solved the cubic and then tried to guess which were the relevant roots. Only the very strongest students made any progress in part (c).

Question 4
This question was relatively unpopular. In part (a), the definitions given for the moments of inertia showed widespread confusion, particularly over the role of the density and the range of integration. Parts (b) and (c) were almost entirely bookwork and were answered well by those who had revised it properly.

Question 5
This question was very popular and was done well. Although there is potential for numerical errors (primarily in (a)(i)), most candidates' calculations were accurate. Part (a)(i) asks candidates to explain carefully what they did in the first step of the algorithm – some candidates gave no explanation. It was quite common for candidates to lose marks in (a)(ii) by not correctly justifying the uniqueness of the solution in (a)(i). Parts (b) and (c) asked about variants of the original problem and many candidates were able to do these well by considering their final simplex tableau from (a) (rather than by re-doing all of the simplex calculations, which some candidates did, which is unnecessary).

Question 6
A popular question. In (a), many scripts proved the "if" or "only if" part, but not both. Part (b) was then largely well done, although few scripts recognised the importance of 13 and 101 being coprime in finding the general solution. In part (c), the intention was to treat k as $1 - 2z$ and apply the previous part. It then turned out that $x + y + z$ equals $-27 + (55z - 88m)$ where m is an integer. In this case $55z - 88m$ can be an arbitrary multiple of 11 and so the closest possibility is when $55z - 88m = -22$ and the desired minimum is 5. Quite a few scripts shunned this direct route for sometimes ingenious calculations which largely also yielded the correct answer.

Question 7
An unpopular question, though very well done by a few. Part (a) was largely well done. Part (b) was best done by considering the Taylor expansion of $f(c)$ centred on x_n; some candidates became confused in part (b) by instead centring on c or even applying Taylor's Theorem to $g(x) = x - f(x)/f'(x)$. Throughout part (b) most scripts missed stating one or other hypothesis when justifying their conclusions.

Paper: Mathematics V

Question 1
The question was popular with many good attempts. The statement of the divergence theorem was generally complete. Some candidates did not define the direction of the area element in the surface integral. Most made a

good attempt at the verification of the divergence theorem. Almost all who attempted it gave a correct potential in the final part.

Question 2
Many candidates struggled to evaluate the two line integrals in the first section. Nearly all of those who attempted this question successfully proved the identities in part (b). Most students spotted the substitutions into the divergence and Stokes theorems in part (c).

Question 3
This questions was less popular than the other two questions in this section. The first part of the question required a modification of the standard spherical polar coordinates (scaling x, y and z by a, b and c respectively) to give a modified Jacobian resulting in the given expression for the volume. The surface area follows from the above modification to the standard spherical polar coordinates and the standard expression for the surface area in terms of the above parameterisation.

Question 4
A popular question attempted by nearly all of the candidates. The bookwork in part (a) was very well done although many lost a mark for inaccurate use of the heat flux. Part (b) was also very well done although many lost marks for not dealing with all of the possible cases in (i) or for stating or deriving incorrectly the Fourier coefficient in (ii).

Question 5
A popular question attempted by the majority of candidates. The bookwork in parts (a) and (b) was very well done. While there were many perfect solutions to part (c), a large number of the candidates were unable to make progress or wasted effort solving separately for $y(x, L/2c)$, $y(x, L/c)$ and $y(x, 3L/c)$, rather than for $y(x, t)$ for all $t > 0$.

Question 6
An unpopular question attempted by about one in five of the candidates. The bookwork in parts (a) and (c) was very well done when attempted. The majority of candidates gave (incorrectly) a uniqueness proof for part (b).

Question 7
Part (a) and the first part of (b) were generally done well, with most students correctly arguing Gauss's Law using the divergence theorem. Few students were able to gain full marks on the second part of part (b) and only a very small number of students stated that the Gaussian surface needed was a sphere. Very few students gave any argument as to why the gravitational field is constant on this Gaussian surface. Success in part (c) was mixed. A large number of candidates made a good attempt at finding the field strength using Gauss's law. Again, few candidates stated the Gaussian surface being used was a sphere. A common problem for many candidates was evaluating

the mass contained within a Gaussian sphere of radius $R > r$. In many cases the change in the form of the mass density at $r = R$ was not taken into account and the exponentially decaying density was integrated between 0 and r. In nearly all attempts, once an expression for the gravitational field was found the candidate then, unnecessarily, proceeded to calculate the potential.

F. Comments on performance of identifiable individuals

The IBM prize was awarded to the top 2 candidates.

G. Names of members of the Board of Examiners

- **Examiners: Prof Lackenby (Chair), Dr Baker, Dr Earl, Dr Howell, Dr Laws, Dr Oliver, Dr Papazoglou, Dr Reid-Edwards.**
- **Assessor: Dr Lipstein.**