Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2013

November 13, 2013

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2013	2012	2011	2010	2009	2013	2012	2011	2010	2009
70–100	49	56	55	50	45	31.21	33.73	33.33	32.89	29.61
60–69	71	78	79	72	69	45.22	46.99	47.88	45.39	47.37
50–59	32	28	23	20	29	20.38	16.87	13.94	13.16	19.08
40–49	4	2	7	10	9	2.55	1.2	4.24	6.58	5.92
30–39	1	2	1	0	0	0.64	1.2	0.61	0	0
0–29	0	0	0	0	0	0	0	0	0	0
Total	157	166	165	152	152	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 157 take the set of four papers AC1, AC2, AO1 and AO2. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
AC1	157	53.92	12.64	65.29	9.52
AC2	157	68.31	15.10	66.54	10.82
AO1	157	61.15	12.98	64.27	9.21
AO2	157	71.40	13.60	65.97	10.91

B. New examining methods and procedures

None

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

The new Prelims course was introduced this year, so that next year will see the revised Part A.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 22nd November 2012 and the second notice on the 1st May 2013.

These can be found at https://www.maths.ox.ac.uk/notices/undergrad/2012-13/part-a, and contain details of the examinations and assessments. The course Handbook contains the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at http://www.maths.ox.ac.uk/notices/undergrad.

Part II

A. General Comments on the Examination

The examiners would like to express their gratitude to

- Nia Roderick for overseeing Part A examinations during 2012/13.
- Also Waldemar Schlackow, assisted by Helen Lowe, for continuing to develop the examinations database, responding to examiner requests and providing such a good framework for the examinations data.
- We would also like to thank Charlotte Turner-Smith, Sandy Patel, Vicky Archibald and Jessica Sheard for all their sterling work in keeping track of the scripts and marks and everything else they do during the busy examination period.
- We also thank those assessors who set their questions promptly, took care in checking and marking them, and met their deadlines. This is invaluable help for the work of the examiners.
- All the assessors and the internal examiners would like to thank the external examiner Dr Mark Wildon for his careful reading of the draft papers, scrutiny of the examination scripts and insightful comments throughout the year.

Timetable

The examinations began on Monday 17th June at 9.30am and ended on Thursday 20th June at 12.30pm.

Medical certificates and other special circumstances

See Section F

Setting and checking of papers and marks processing

As it is usual practice, questions for AC1 and AC2 were set by the examiners and also marked by them. The papers AO1 and AO2 were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from the teaching committee in order to minimize the need for recalibration.
The internal examiners met in December to consider the questions for AC1 and AC2. The course lecturers were invited to comment on the notation used and in general on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to external examiner.
In a second meeting the external examiners discussed the comments of the external examiner and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term courses and at the end of Hilary and beginning of Trinity term for the Trinity term courses. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.
All examination scripts were collected by the markers from the Mathematical Institute and returned there after marking. A team of graduate checkers under the supervision of Nia

Roderick, assisted by Sandy Patel, sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. Here 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.
The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
AC1	65	41	19
AC2	80	53	24
AO1	75	48	23
AO2	83	60	27

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
90	1	1	0.64
86	2	2	1.27
85	3	3	1.91
83	4	5	3.18
81	6	8	5.10
80	9	10	6.37
79	11	13	8.28
78	14	14	8.92
77	15	16	10.19
76	17	18	11.46
75	19	26	16.56
74	27	31	19.75
73	32	33	21.02
72	34	38	24.20
71	39	41	26.11
70	42	49	31.21
69	50	54	34.39
68	55	63	40.13
67	64	73	46.50
66	74	79	50.32
65	80	87	55.41
64	88	95	60.51
63	96	104	66.24
62	105	110	70.06
61	111	115	73.25
60	116	120	76.43
59	121	130	82.80
58	131	132	84.08
57	133	139	88.54
55	140	141	89.81
54	142	142	90.45
53	143	148	94.27
51	149	149	94.90
50	150	152	96.82
48	153	155	98.73
41	156	156	99.36
35	157	157	100

B. Equal opportunities issues and breakdown of the results by gender

Table 5, page 6 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

Range	Total		Male		Female	
	Number	%	Number	%	Number	%
70–100	49	31.21	40	34.78	9	21.43
60–69	71	45.22	54	46.96	17	40.48
50–59	32	20.38	18	15.65	14	33.33
40–49	4	2.55	2	1.74	2	4.76
30–39	1	0.64	1	0.87	0	0
0–29	0	0	0	0	0	0
Total	157	100	115	100	42	100

C. Detailed numbers on candidates' performance in each part of the exam

Table 6 (and continued in Tables 7 and 8) give the detailed performance of candidates' on each question of the exam.

Table 6: Analysis by question

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AC1	Q1	5.85	5.85	2.29	157	0
AC1	Q2	5.35	5.35	2.42	155	0
AC1	Q3	5.35	5.35	2.68	155	0
AC1	Q4	7.76	7.76	1.63	156	0
AC1	Q5	6.57	6.57	2.24	156	0
AC1	Q6	7.26	7.26	1.95	156	0
AC1	Q7	7.46	7.46	2.03	155	0
AC1	Q8	3.75	3.75	2.09	154	0
AC1	Q9	5.06	5.06	2.80	155	0

Table 7: Analysis by question

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AC2	Q1	14.94	15.60	5.94	117	7
AC2	Q2	15.26	18.53	8.48	17	6
AC2	Q3	15.63	16.07	6.06	59	6
AC2	Q4	16.55	18.24	5.07	17	5
AC2	Q5	17.43	17.60	4.25	101	3
AC2	Q6	16.30	17.85	5.84	48	9
AC2	Q7	7.43	9.5	4.73	18	22
AC2	Q8	18.67	18.77	4.47	149	1
AC2	Q9	15.63	16.91	7.06	102	11
AO1	A1	5.47	5.58	2.27	64	4
AO1	B1	8.03	8.03	1.39	39	0
AO1	C1	6.72	6.98	2.95	91	5
AO1	D1	7.09	7.18	2.10	89	2
AO1	D2	6.68	6.71	2.06	84	1
AO1	E1	5.73	5.78	1.90	83	2
AO1	E2	6.08	6.56	3.11	54	8
AO1	F1	7.87	7.87	2.50	15	0
AO1	G1	6.35	6.46	2.56	113	5
AO1	H1	5.64	5.64	2.60	61	0
AO1	J1	7.32	7.44	2.01	88	2
AO1	K1	5.81	5.81	1.60	72	1
AO1	K2	5.73	5.88	2.92	40	4
AO1	M1	6.96	7.01	2.27	116	2
AO1	M2	4.82	5.28	3.19	88	12
AO1	O1	7.74	7.74	2.17	80	0
AO1	O2	7.14	7.27	2.38	75	2
AO1	P1	9.14	9.14	1.43	74	0
AO1	P2	8.31	8.31	1.72	77	0

Table 8: Analysis by question, continued

Paper	Question Number	rawAvg	Avg Used	Std Dev	Used	Unused
AO2	A2	14.65	15.5	4.61	18	2
AO2	B2	13.52	14.79	6.00	19	4
AO2	C2	19.18	19.75	4.86	63	4
AO2	D3	15.73	17.11	6.21	27	6
AO2	D4	17.83	18.09	4.09	22	1
AO2	E3	15.23	17.03	5.39	32	7
AO2	E4	17.87	18.22	5.48	36	2
AO2	F2	21.00	21.00	5.23	4	0
AO2	G2	11.24	13.75	6.05	24	10
AO2	H2	12.75	12.75	3.86	4	0
AO2	J2	11.06	13.09	5.11	11	5
AO2	K3	18.08	18.10	4.10	60	1
AO2	K4	17.74	18.44	5.43	39	3
AO2	M4	12.63	13.74	5.39	43	9
AO2	M5	17.14	18.48	5.69	61	8
AO2	O4	20.35	20.86	5.00	59	3
AO2	O5	19.22	19.53	4.28	36	1
AO2	P3	14.47	16.22	6.14	41	8
AO2	P4	20.93	21.17	4.25	29	1

D. Recommendations for Next Year's Examiners and Teaching Committee

None.

E. Comments on sections and on individual questions

The following comments were submitted by the assessors.

AC1

Algebra

Question 1: [Dual spaces] In part (a), many candidates omitted to comment on why V' is a vector space. Generally, the question seemed reasonable.

Question 2: [Adjoints/ projections] In part (b), many solutions showed only one inclusion for $U = W^{\perp}$. Otherwise, it seemed OK.

Question 3: [Euclidean domains] In part (b), many solutions were incomplete: sometimes the existence of such d was not explained, and some solutions did not say why the remainder term was in I. But again, the question seemed reasonable.

Analysis

Question 4: Parts (a)-(c) of this short question were very straightforward and were mostly answered well. Most used the Cauchy–Riemann equations for (b) (sometimes losing marks by forgetting to check continuity of the partial derivatives); some used the definition of a holomorphic map, which was faster. Those who realised that they could use (b) as well as (c) to answer (d) finished the question very easily; others gave more complicated answers to (d), only some of which were correct. Many looked for a Möbius transformation taking three points on the line bounding U_1 to three points on the circle bounding U_2, without noticing that 0 should also be sent to ∞.

Question 5: This question was very similar to one on a problem sheet. It was mostly quite well done, although lots of candidates used a contour with an unnecessary indentation around the origin, which made the question longer than it should have been. Many also did not spot the relationship between the final integral and the real part of the preceding integral.

Question 6: Many candidates found the bookwork part of this question more taxing than the straightforward application, but on the whole it was well answered.

Differential Equations

While there were 10s scored on all of these problems, with one candidate scoring 30 on the three, these were evidently lower-scoring than in recent years.

Question 7: A question built around Picard's Theorem, this was generally well-done; the definition of the norm made difficulties for some candidates but most candidates seemed to get the idea quite rapidly.

Question 8: A question on the classification of second-order linear PDEs, this was the least well-done of the three. Quite a few candidates could not do the classification correctly; most did not find that the solution from $y < 0$ vanishes on the axis and so could not find the solution in $y > 0$.

Question 9: A question on Laplace transforms; a very common error was to confuse 'the derivative of the Laplace transform' with 'the Laplace transform of the derivative'.

AC2

Algebra

Question 1: [Primary Decomposition Theorem] This was the most popular question, the large majority of candidates chose it. Part (d) seemed to be the most difficult part (fortunately the typo did not cause confusion). Generally, there were many good solutions.

Question 2: [Inner products, Bessel Inequality] Very few candidates did this question.

Question 3: [Chinese Remainder Theorem and applications] This also was a popular question, and there were many good solutions. Though, in part (b) several candidates first proved that the kernel of the relevant map defined on R was $I \cap J$, and subsequently tried to prove that the map was one-to-one.
The second half of part (d) was probably the most difficult part.

Analysis

Question 4: This question was not popular; part (d) looked off-putting, although in fact with the hint it was very straightforward. Only a few candidates successfully completed (e) by subdividing an arbitrary triangle in U into triangles completely contained in either the closed upper half plane or the closed lower half plane, approximating these by triangles in U_+ or U_- and using (b), (d) and (c).

Question 5: This was the most popular of the AC2 Analysis questions. Some candidates gave very good and complete answers though most were able to answer some parts well but not others. Many got bogged down in estimates in (d), and with hindsight it might have been helpful to ask for $g(z)$ to be rewritten as $\pi(e^{2\pi iz}+1)/(e^{2\pi iz}-1)$ and to ask for a bound valid where $\mathrm{Im}(z) \geq 1$, as well as stressing the periodicity of $g(z)$ and the fact that $g(-z) = -g(z)$.

Question 6: This question was less popular than Question 5 but was well answered by many candidates. It was however disturbing to see how many believed that in order to show that a function f is constant, it is enough to prove that $g \circ f$ is constant where g is not constant.

Differential Equations

Question 7 on characteristics, was attempted by relatively few students, less than four dozen. The first part of the question was done quite well by most of the students. The core of the question was devoted to the determination of the domain of definition (located between $y_1(x) = |lnx|$ and $y_2(x) = \sqrt{1 + ln^2 x}$), obtained in several steps. Many students found the upper bound, and some part of the lower bound, either for $x < 1$ or for $x > 1$, but none of them put all elements together to obtain the full answer. Overall, answers were satisfactory. The most common error was to forget that along characteristics all three coordinates are functions, thus the anti-derivative of $sz(s, t)$ isn't $s^2 z(s, t)/2 + c$ for example, but this wasn't that frequent.

Question 8 on phase planes, was taken by many students, more than 80% of the cohort. It turned out that it posed very little difficulty to most of the students who took it, and a lot of them obtained high marks. The bulk of the work was to classify the various critical points. Apart from calculations mistakes, and confusions between the definitions of the various types of nodes, this was done well. The last question was rarely done very well: in particular a small number of students drew sketches compatible with the null-clines : this was far more relevant for a sketch than the precise evaluations of the asymptotes of to the trajectories near saddles.

Question 9 on Green functions, was also quite popular, and chosen by a large proportion of the cohort, and done well by most of them. It was difficult to do the question well if it was not started correctly, and the computation of the Green function sabotaged by silly mistakes (typically, $exp(x^2) \times exp(x^2) \neq exp(x^2)$). The proof of the weak and strong maximum principle (to call the positivity property by its name) was often done well for the weak part, and less well for the strong part. It was an easy question as well, as the marks were relatively high on average.

AO1 and AO2

A: Introduction to Fields

A1: Across the various Honour Schools there were approximately 95 attempts at this question. Almost all candidates earned full marks for the definitions in Part (a). In Part (b) however a small number of candidates failed to read the question properly and sought to prove the Tower Theorem; a not so small number lost marks by failing to do the relevant elementary linear algebra from Mods correctly.
In Part (c) candidates who failed to include the converse of (b) (that if $|K : F|$ and $|L : F|$ are finite then so is $|L : F|$) forfeited a mark. What was expected in the last part was something reasonably clear along the lines: the construction of a new point using ruler & compass involves finding the point of intersection of two lines, of a line and a circle, or of two circles; if the coordinates of known points or lengths of lines already constructed lie in a field F then the coordinates of a new point are obtained by solving linear or quadratic equations and therefore lie in an extension E of F of degree at most 2; consequently coordinates of constructible points lie in iterated quadratic extensions of $\mathbb{Q}$ contained in $\mathbb{R}$; by the Tower Theorem the degree of such a field extension of $\mathbb{Q}$ is 2^m for some natural number m and therefore (by the Tower

Theorem again) the coordinates of any constructible point are algebraic numbers of degree a power of 2; to double a cube one would need to construct a line of length $\sqrt[3]{2}$, of which the degree over $\mathbb{Q}$ is 3, and since 3 does not divide any power of 2 this is not possible by ruler & compass methods. Disappointingly few candidates related the geometry to the algebra. Very few realised that the word 'coordinates' must be relevant and spoke of adjoining points to a field, a category error that could have been avoided. There were, however, a few excellent answers.

A2: There were 26 answers to this question. The first part was generally well done, though a few candidates used unnecessarily sophisticated machinery to show that there must be just three irreducible monic polynomials of degree 2 over $\mathbb{F}_3$ before identifying those three. The second part was also well done. Most, but not quite all, candidates came to grief on the third part, however. One candidate, but fortunately only one, thought that $e^{2\pi i/5}$ is a primitive + fifth root of unity in $\mathbb{F}_{81}$. Only three candidates saw that it would be a good idea to investigate a^{16}; unfortunately, two of these made the unwarranted assumption that a must be a generator of the multiplicative group $\mathbb{F}_{81}^{\times}$. One candidate came very close with an element whose 5^{th} power turned out to be -1, so that it was a primitive 10^{th} of unity.

B: Group Theory

B1/B2: The short question was attempted by 51 candidates, who generally did very well on this easy question, scoring an average of 7.9 points. One standard way to drop a point was in the last part, giving insufficient argument for why there were only two possible configurations. By contrast, the long question was found surprisingly hard, with 27 attempts at an average of only 13.3 marks (53%). There were several candidates who attempted the easy bookwork part and gave up when the going got rough, but even among those candidates who stayed with the question, there was a wide range of levels of understanding of the orbit structure of a group action.

C: Number Theory

I would like to make one general remark. I think it would be very helpful for the students to know how many marks each section of the question receives. There were some scripts that wasted a lot of time and effort to parts of the question that were only going to attract 1 or 2 marks. It would have been clear to them if they would have known what the total mark for that particular part was.

C1: There were 131 scripts. 36 of these attracted marks of 4 and below; 76 attracted marks of 7 and above.
The exam consisted entirely of material that had been seen before. Some students had not properly revised the material or assumed that the material of the last lecture (RSA) was not important for the exam. Many proofs of Euler's Theorem were unnecessary lengthy.

C2: There were 91 scripts. 9 of these attracted marks of 10 and below; 53 attracted marks of 20 and above; 67 marks of 18 and above.

Part (a) was generally well-done but some candidates did not remember the formula for the Legendre symbol $(2/p)$. All in all 9 marks were assigned to this part. Part (b) was the most challenging one though it only attracted 6 marks. Some students missed the point of the question that the QRL was to be extended to non-primes. The proof needs to use the QRL for primes and the fact that modulo 2 $(ab-1)/4 = (a-1)(b-1)/4$. Part (c) was generally done well with candidates being pleasingly fluent in the different rules and tricks when calculating the Legendre and Jacobi symbols. This part attracted 10 marks.

D: Integration

The standard was generally very respectable. Most candidates were prepared to try to justify their arguments according to the spirit of the course. Only a few appeared to be really weak, and here were some excellent ones.

D1/D2: Solutions to D1 and D2 were quite strictly marked. In D1, a few candidates tried to use the definition of integrals from the Etheridge notes; in principle this was acceptable, but it was much easier to use the simpler definition given in lectures. The most common loss of marks was for failing to specify that the integral of the absolute value, or the positive and negative parts, of g must be finite. In both questions, calculations were mostly correct; measurability was mentioned in more solutions than usual, although many still lost a mark for ignoring it.

D3/D4: D3 and D4 were not very popular. This was surprising for D3 which was a question of a standard form, although the very last part required candidates to treat the cases $\beta < 1/2$ and $\beta > 1/2$ separately (a few did so). D4 was a non-standard question, involving topics which were close to some lectures near the beginning of the course and some near the end. The Lebesgue space L^1 appeared for the first time in an examination at this level. A few candidates saw that Fatou's lemma should be invoked to justify the very final part.

E: Topology

E1: This question successfully assessed the students' understanding of the product topology and of the notion of a basis for a topology. The majority of students were able to answer parts (a), (b)(i) and (b)(ii). Fortunately, only very few students asserted in these parts that every open set in the product topology is of the form $U \times V$, which has been a common error in previous years. But many found (b)(iii) challenging, and there were many attempts at counter-examples, whereas the question has a positive answer.

E2: Many students find the quotient topology rather challenging, and so this question was comparatively difficult. In part (b), the goal is clearly to find a continuous surjection from S^2 to Y, but unfortunately, many students found this difficult. When students found the correct equivalence relation but without fully accurate reasoning, I gave considerable partial credit.

E3: This question has a more analytic flavour than most questions on this course. The first parts were fairly straightforward bookwork, and were generally very well done. But these were counterbalanced by some more challenging later questions. The final one is quite difficult. It is best attempted by using the observation that a real-valued sequence is convergent iff it is Cauchy. There were a few really outstanding solutions.

E4: This question was comparatively long, but it was very popular, possibly because it had inviting and accessible parts throughout it. Unsurprisingly, (c)(iii) was found to be the most challenging. Its solution required the use of both (c)(ii) and (b). I was particularly impressed by the large number of high quality attempts at this question.

F: Multivariable Calculus

F1: A straightforward question on total derivative, but with quite a few parts, leading up to a question on the Inverse Function Theorem, this was generally well-done. The most common error was in finding the constant in (b)(ii) but this only carried one mark so there were a lot of 9's.

F2: A question on total derivative and Lagrange Multipliers this attracted only four attempts; however three of these were impressively sophisticated.

G: Calculus of Variations

G1: This question was a popular choice. Almost all candidates showed good understanding of the basic steps involved in deriving the Euler-Lagrange equations. However, many were confused about applying them to the case of two independent variables, many using a single parameter α to govern variation in both y and z, and some a single test function η for both y and z. The concept of natural boundary conditions was generally understood and applied satisfactorily.
Almost all candidates had the right idea about how to obtain the Euler-Lagrange equations for the problem. Those who were at home with constant-coefficient equations could easily write down the solution, and they typically obtained full marks for the question. Too many, however, made elementary errors of sign which gave rise to the wrong differential equations. Many also made very heavy weather of solving the equations.

G2: Virtually all candidates could start with correct Euler-Lagrange equations, and most could see the constancy of L from the Beltrami identity. Explaining why $\sqrt{L}$ generates the same equations was skipped by most candidates. When addressing the geodesics on the surface of revolution, many candidates went back to the formulation with the parameter t, instead of realising that the point of the preceding work had been to eliminate t. This would have been all right if they had checked the *two* equations that then arise, and realised that this method was providing extra information about $\eta(t), \zeta(t)$ (i.e. constant speed on the surface). Instead, they got lost. Very few simply wrote down the Euler-Lagrange equation (for $\zeta(\eta), \eta(\zeta)$ in the two cases), and checked that it was satisfied by the stated solutions.

H: Classical Mechanics

H1: A surprising number of candidates started with a K.E. calculation which ignored the nature of the rod and treated it as a point mass. However, the concept of the Lagrangian method was well understood. Virtually all candidates showed a clear understanding that the constancy of $T + V$ had to be properly derived from the form of the Lagrangian, and everyone saw the role of the ignorable coordinate ϕ. Very few candidates, however, were able to describe the motion with constant θ.

H2: Only four candidates attempted this question. The bookwork was reasonably well learnt and reproduced, but the rigid body in the last part of the question defeated them all. There was no attempt to break the problem down by using the Parallel Axes theorem.

J: Quantum Theory

J1: This question required candidates to show that the given Gaussian wave packet satisfies the free Schrodinger equation. A similar question appears on the "Additional questions" problem sheet. Most candidates correctly wrote down the time-dependent Schrodinger equation, and gave a correct definition of stationary state, although a common error was to give the wrong explanation for the Gaussian wave packet not being a stationary state. Many candidates correctly computed the partial derivatives to conclude that the potential is zero, although factors often went missing.

J2: Many attempts at this question were somewhat disappointing. The question essentially combines two questions from the problem sheets, involving a 3d harmonic oscillator potential, with a constant background electric field also present. Part (a) of the question requires both separation of variables and completing the square, which then allows one to use the given one-dimensional result. This part of the question was on the whole very well done, although only a small number of candidates computed the correct degeneracy (despite this being on a problem sheet). Part (b) was only well-answered by one candidate. I think many candidates simply got a bit lost, not realizing that they had to return to the one-dimensional oscillator to determine the form of the first few (non-normalized) Hermite polynomials (which should at least be familiar from lectures). Part (c), when attempted, was generally answered well.

K: Fluid Dynamics and Waves

K1: The first part was mostly done well by all candidates, though some left the RHS of the equation giving $d\bar{c}/dt$ for two general points c_1 and c_2 instead of specialising to c and $-c$.
Only a handful of candidates thought to introduce polar coodinates for the second part. One otherwise perfect solution was sadly marred by a (very rare) confusion over the sign of the point source. I gave substantial partial credit for candidates who formulated equations for dy/dx by eliminating t, especially if they knew that $y = xv(x)$ was then the right substitution. One candidate successfully computed an algebraic relation between y and x for the trajectories, aptly commenting that it was "not very nice".
Some candidates, perhaps confused by the letter c, though I used this notation in lectures, treated the RHS of $d\bar{c}/dt$ as constant and obtained straight line trajectories. Others formulated equations for dx/dt and dy/dt that held only when c was real. I gave substantial

partial credit to candidates who obtained the correct expression for $r(t)$ even if they thought they had found $x(t)$, and to one candidate who drew qualitatively correct sketches in lieu of calculations.
Many candidates could guess/intuit the correct answer to the final part without finding the trajectories.

K2: This was generally well done, though one candidate had the phase and group velocities the wrong way round. I was overly generous in assigning 4 marks to computing the group velocity. Almost all candidates gave a bare formula with little attempt to intepret or simplify it as preparation for the next part.
Most candidates verified that the stationary point was a minimum, either by computing the second derivative or by global arguments (as I expected) so I only gave partial credit to those who only solved for dc/dk=0.
I was generous to candidates with overly complicated (but correct) expressions for the group velocity at the critical wavenumber. Many did not collect powers of the different variables, and some lost one of the two nested square roots.
In the last part, a number of candidates had the two cases the wrong way round, possibly through guessing the likely answer in lieu of calculation, and some thought the wave packet expanded or contracted in the two cases.

K3: The proof of Blasius' theorem was done well by almost all candidates, though quite a few had trouble with the overall sign, and some had the sign for the pressure wrong in Bernoulli's equation. A few proved the latter, although this was not expected (perhaps this should have been stated, as it was in question K4). Some candidates asserted that $dw = d\overline{w}$ on ∂B without explanation, for which I deducted a mark. Most at least wrote that $\mathrm{Im} w = \psi$ was the streamfunction, and so constant on ∂B.
For the second part, most candidates wrote down $w = (U - iV)z - i\Gamma/(2\pi)\log z$ without realising that this was just the far-field asymptotic behaviour. Some assumed the body was a circle and used the Milne-Thompson circle theorem. Only a minority produced an argument equivalent to the proof of the Kutta–Joukowski theorem given in lectures, deforming the original contour ∂B to a large circle and using the above asymptotic form of w for large z.
I gave roughly half marks (out of 7 available for this part) to candidates who correctly evaluated Blasius' integral for $w = (U - iV)z - i\Gamma/(2\pi)\log z$ around ∂B.
A few candidates had serious difficulty with evaluating the contour integral using residues.
A few candidates wrote down correct formulae for the lift and drag without calculation.
A substantial minority did not realise that "lift" and "drag" are defined relative to the on-coming flow. Some ended their answer with expressions for F_x and F_y, or calculated the forces parallel and perpendicular to the body instead.

K4: There was regrettably a mistake in the question. The T should have been T/ρ in the third boundary condition. One candidate correctly pointed out the dimensional inconsistency of the given expression, and a few gave the correct expression with T/ρ without comment. The majority arrived at the expression in the question, with some noting that they had set $\rho = 1$ to achieve it.
I was looking for some mention of curvature for the "geometrical intepretation" in the first part.

Most candidates derived the linearised boundary conditions correctly. My original mark scheme assigned 2 marks to each step, making 12 in total. Where applicable I gave 12 marks overall, rather than 2 marks at each intermediate stage. Some candidates ignored the U^2 term when linearising Bernoulli's equation without choosing a suitable $F(t)$ to cancel it. A few candidates asserted that the pressure must be continuous across the interface despite having been given information to the contrary in the question.
Most candidates wrote down correct functional forms for $\phi_{\pm}$, but many made calculational errors with minus signs and powers of k when combining the three boundary conditions into a dispersion relation. A few candidates left e^{-ky} expressions in their boundary conditions, forgetting that they were applied on $y = 0$.
Some candidates found ω but did not write down an expression for the phase speed $c = \omega/k$. Many candidates correctly observed that the flow became unstable when $U^2 > 2(T/\rho)k$, but few commented on the mean of the two phase speeds being $U/2$, the same as the mean of the background fluid velocities.

M: Probability

M1: Reasonable answers on the whole. In part (c), too many candidates took the probability of the union of two events to be the sum of the two probabilities, even though the events are not disjoint. Units (variously chosen to be minute/5 minutes/15 minutes/hour) caused confusion at times; for full credit in (d), units should be specified somewhere.

M2: This question caused more difficulty than intended. Parts (a) and (b) were question 4 of the first problem sheet (on the problem sheet, the distribution in part (b) was given). In (b), a fair few candidates argued along the lines that from (a), $1 - M_n$ converges in probability to 0, hence $1 - M_n$ converges in distribution to 0, hence $n(1 - M_n)$ converges in distribution to $n \times 0 = 0$.
For part (c), the central limit theorem says that the distribution of S_n concentrates around its mean, and that the typical deviations of S_n are of order $\sqrt{n}$. The presentation here was unfamiliar though, and many candidates did not succeed. Many gave the central limit theorem more or less correctly, but couldn't go from the fact that $\frac{S_n - n/2}{cn^{1/2}}$ converges in distribution to seeing that $\mathbb{P}\left(S_n < n/2 + n^{1/2}\right)$ has a non-degenerate limit.

M4: This question was not well answered on the whole. In part (d), many candidates arrived correctly at answer $\sum k\alpha_k < \infty$, but when asked to interpret this, didn't observe that $\sum k\alpha_k$ is simply the mean of the distribution (α_k).
The calculation in part (e) was generally done well.
Good answers to part (f) were rare. The mean return time m_1 is twice the mean of a geometric(1/2), giving $m_1 = 4$, so in the stationary distribution $\pi_1 = 1/4$. The result that $p_{11}^{(n)} \to \pi_1$ applies only for aperiodic chains. Quite a few candidates asserted that the chain was aperiodic, even though this contradicted their own answer to (c); here $p_{11}^{(n)}$ is 0 whenever n is odd, so that 0 is one limit point of the sequence. For the other, consider the two-step chain $X_0, X_2, X_4, \ldots$. This chain *is* aperiodic, and state 1 has stationary probability 1/2.

M5: This question posed few problems for well-prepared candidates. The general standard of answers was high. In (b)(i), too many candidates still can't reliably arrive at $\mathrm{Var}(S_n) = n\,\mathrm{Var}(X_i)$. In (b)(ii), writing down the first few terms and comparing them numerically was not convincing enough; it's not hard to write the general term. Part (b)(iii) involved assembling a few different steps, but the ideas were familiar from an example in lectures and many succeeded well.

O: Statistics

O1: Some answers gave the "expected information" as something that depended on $(X_1, \ldots, X_n)$ or $(x_1, \ldots, x_n)$ – both of these are errors. However, most candidates made good attempts at the whole question and got good marks.

O2: This was a question on material that was new in the course this year (Bayesian inference). Most candidates were comfortable with this material and did the question well. Some lost marks in (a) by finding the posterior for a sample of size 1, rather than for a sample of size n.

O4: Many candidates did this question accurately and well, and scored high marks. The most common place to lose marks was when sketching the power function in (e).

O5: Most candidates did (a) and (b) well. In (c) a common error was to get the degrees of freedom wrong for the modified χ^2-test. Part (d) is to test whether or not a Poisson model is a good fit to the data given. Some candidates ended by saying things like "we do (or do not) want to reject H_0," but did not make it clear what they meant by this – i.e. is a Poisson model an acceptable fit, or not? (It is.) It is better to finish by saying something like "the fit of a Poisson model is fine" (and if talking about an H_0, to say what that H_0 means). In some answers it was unclear whether the p-value had been identified as 0.82 or as 0.18.

P: Numerical Analysis

These two short problems proved quite manageable for the students, with 25 out of 103 getting full points of 10+10.

P1: (a) Almost all students knew the formula. (b) Most could prove uniqueness. (c) Many could write down the correct formula for the weights

P2: (a) Almost all students found the upper-triangular factor correctly. (b) Most found the lower-triangular factor correctly. (c) Most wrote down a correct counterexample, though usually more complicated than necessary. In a fair nuber of cases they had some difficulty producing a good *proof* that that there could be no LU factorization of this matrix.

These two longer problems provided a good range of scores. Overall, P3 proved a bit harder than P4, though more students attempted it.

P3: (a) Many defined orthogonality without mentioning anything about pk being a polynomial of degree k; these got 2/5 points. (b) Many people got the right answer on this calculation (sometimes after many pages of figuring). (c) A good number got this right, though just a minority set up the induction argument with real clarity. It was startling, however, how many people utilized one or the other of two elementary falsehoods: (1) if a function f is not even, then f must be odd; if the integral of a function f is zero, then f must be odd.

P4: (a) Everybody got this. (b) Almost everybody got this. (c) Many got this, though quite a few confused matrices Q and vectors qj. (d) Many got this too, the hardest part of the problem. (e) People did well on this.

F. Comments on performance of identifiable individuals

Removed from public version.

G. Names of members of the Board of Examiners

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  Prof. K.P. Tod (Chair)
  Dr Y. Capdeboscq
  Dr K. Erdmann
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  Prof. G Reinert
  Dr M. Wildon (External Examiner)

- **Assessors:**
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