Oxford Mathematical Institute

March 2003 Issue 1

Newsletter

Inside this issue

On the move

Our Institute in Oxford houses a large and brilliant community of mathematicians. Their achievements do not often attract the attention of the media since it is not in the nature of mathematicians to practise immodesty and it is not in the nature of journalists to inquire too deeply into mathematics. But over the decades, and indeed the centuries, the work of Oxford mathematicians has had a profound impact on our society.

This Newsletter tells our friends, and our former students and colleagues, something of the successes and achievements of the past year.

Success, however, brings its problems. Our current building, which seemed so spacious when it opened in 1966, is now hopelessly overcrowded. Our colony in Little Clarendon Street gradually expands as we take over more floors of Dartington House, but we desperately need a new building in which the whole community – professors, lecturers, post-docs, and graduate students – can work together in comfort. We also need facilities to draw undergraduates more fully into the life of the department. Informal interaction is the lifeblood of mathematics, and it is greatly inhibited by scattering people over different colleges and University buildings.

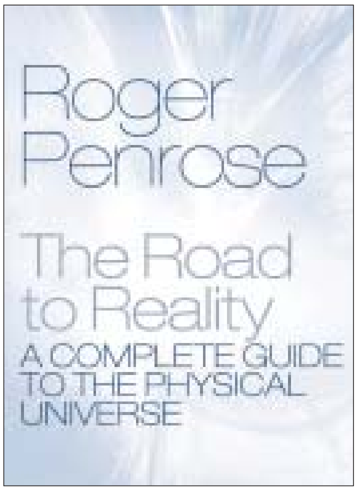
In January of 2002, our External Advisory Panel set us down the road of planning a new building for the whole department. It is early days yet, but we hope to see these tentative first steps come to fruition over the coming years. Subsequent issues of the Newsletter will report progress.

N. M. J. Woodhouse
Chairman, Mathematical Institute

Meeting of minds

Mathematics, like music, is a language all its own and one that crosses national boundaries. Nowhere is this seen more clearly than at international conferences and this year in Beijing was no exception when China played host to 3000 mathematicians from around the world for the four-yearly International Congress, ICM 2002. The Opening Ceremony brought the mathematicians – in a fleet of buses driven through streets cleared of traffic – to the Great Hall of the People where in the presence of Jiang Zemin, President of the People's Republic of China, the conference was declared open.

Oxford was well represented: Professor Frances Kirwan was invited to be one of the 20 Plenary speakers, Professor Ulrike Tillmann was also invited to speak. At the Congress Professor John Ball was elected to the four-year Presidency of the International Mathematical Union.



An important new book from **Roger Penrose** is scheduled for publication in spring 2003. Ambitious in scope and scale, *The Road to Reality* is Penrose's picture of the physical universe complete with a masterly survey of the mathematical underpinnings.

'I want to describe as clearly as possible our present understanding of the universe and to convey a feeling for its deep beauty and philosophical implications as well as its intricate interconnections.'

Penrose, Roger *The Road to Reality* Jonathan Cape (2003) 0-224-04447-8

A feature of science today is the gradual erosion of boundaries within and between subjects. This is seen in mathematics too and some of the most exciting of recent research has resulted from the imaginative use of ideas and tools from one branch of the subject in the elucidation of another, apparently unrelated area.

Moduli Spaces of Riemann Surfaces

Frances Kirwan writes about her Beijing paper on Moduli Spaces of Riemann Surfaces.

MARC ATKINS

'A surface in Euclidean 3-space $\mathbb{R}^3$ is easy to visualise; it is typically defined by one equation in three variables x, y and z, just as the unit sphere is defined by the familiar equation $x^2 + y^2 + z^2 = 1$. The crucial property which a surface must have is that *locally* it 'looks like' Euclidean 2-space $\mathbb{R}^2$. We get different kinds of surface depending on our interpretation of 'looks like': a topological surface if our interpretation uses continuous functions, a smooth surface if we use differentiable functions and a Riemann surface (called after the great 19th century German geometer Bernhard Riemann) if we replace $\mathbb{R}^2$ with the complex plane $\mathbb{C}$ and use functions which are differentiable in the sense of complex numbers (i.e., holomorphic functions).

According to a very old joke, a topologist is someone who cannot tell a teacup from a doughnut, the point being that the surface of a teacup is topologically the same as the surface of a (ring) doughnut. In fact, each is topologically a torus, sometimes called a 'sphere with one handle'.

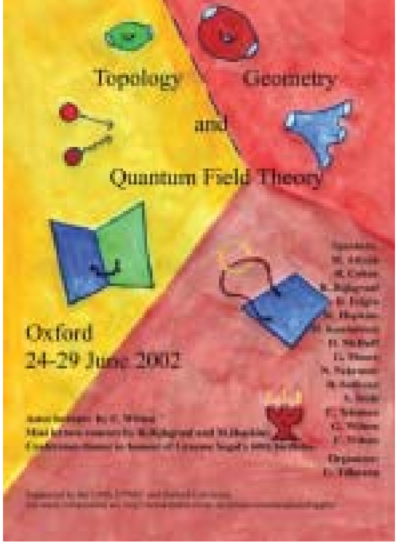
Riemann realised that a topological surface can usually be made into a Riemann surface in lots of different ways (although this is not true for a sphere, which can be made into a Riemann surface in essentially only one way). In fact he observed that if S is a 'sphere with g handles' where $g \geq 2$, then there is a $(3g - 3)$-dimensional family of complex parameters which describe the possible Riemann surfaces which are topologically the same as S. In modern mathematical terminology, Riemann's observation amounts to the fact that the moduli space of compact Riemann surfaces of genus g has complex dimension $3g - 3$ when $g \geq 2$.

Moduli spaces are very beautiful geometric spaces which arise in classification problems in geometry: a moduli space for a given classification problem is the set of equivalence classes of the geometric objects to be classified, itself endowed with a geometric structure which reflects the way the objects can vary geometrically.

Understanding the topology of the moduli spaces of compact Riemann surfaces of genus g has for a long time been an important goal in algebraic geometry, and very recently a significant advance has been made concerning the 'limiting case' when g tends to infinity. In fact this progress has been made via a reinterpretation of the problem in purely topological terms, and it is not algebraic geometers but a group of topologists, including Ulrike Tillmann at Oxford, who deserve the credit for this particular advance.'

Unfortunately Frances was unable to present her paper in person as a result of illness.

Time to celebrate



NATHALIE WAHL

Mathematics and theoretical physics have always been intertwined. It used to be said that advances in physics sometimes made use of pre-existing mathematics, and in turn could influence new mathematical research. Over the past twenty years, however, it has become apparent that ideas from particle physics can provide new techniques for studying traditional problems in mathematics. One such topic is string theory with its links to geometry and topology.

String Theory & Topology

MARC ATKINS

Ulrike Tillmann on the background to her Beijing lecture *String Theory and Topology*

'In classical mechanics a particle moves along a trajectory which minimizes its 'action' – a certain functional of its paths. In Feynman's path-integral approach to quantum mechanics the basic idea is the 'amplitude' for the particle to get from one place to another in a given time, and is calculated as an average over all possible paths from one position to the other, weighted with the exponential of the classical action.

Particle theory only works well when gravity is ignored. In searching for a theory that unifies quantum mechanics and gravity theoretical physicists have invented string theory. The point particle is replaced by a 'string', a little loop. As time evolves the string moves and sweeps out a surface, and while it does this it occasionally splits or interacts with other strings.

To calculate the 'amplitude' of a string it is necessary to take an average – called the path-integral – over all possible surfaces in space-time. Unfortunately the space of all possible surfaces is too large and no rigorous mathematical way of taking such an integral is known. If the theory is conformally invariant, however, it is possible to replace the space of all surfaces by the moduli space of compact Riemann surfaces, as described by Frances Kirwan in her article. Because the moduli space is finite dimensional it is possible to make sense of a path-integral. This is why much recent interest in the topology of moduli spaces has been motivated by theoretical physics.

Since the early sixties, moduli spaces of Riemann surfaces and their variants have been studied mainly by algebraic geometers (with many important contributions by Frances Kirwan). Recently, however, topologists have been able to solve a long-standing, central problem in this area. Using deep, now thirty-year-old results by the Oxford mathematicians Dan Quillen and Graeme Segal, the simple observation that taking the union of two strings α and β is the same as the union of β and α led to a way of associating with the moduli spaces of Riemann surfaces a commutative group (in a fuzzy, topological sense). And thus string theory provided the topologists with a clue as to how to solve a long-standing problem in algebraic geometry. What a marvellous world.'

Topology, Geometry & Quantum Field Theory

Oxford has been very much at the centre of this new interaction of physics and mathematics. The perfect opportunity for Oxford to hold an interdisciplinary symposium was presented by Graeme Segal's 60th birthday. Formerly a Fellow of St. Catherine's and now a Fellow of All Souls, **Graeme Segal** has spent most of his working life in Oxford. He has been central to the mathematics-physics dialogue during the past two decades and many distinguished scientists from both camps were delighted to honour a colleague, teacher, and friend and to attend the symposium and celebration banquet.

Ed Witten (Princeton) gave the first Astor lecture and this was open to all members of the University. Mini-lecture courses by **Mike Hopkins** (MIT) and **Robbert Dijkgraaf** (Amsterdam) produced stimulating dialogue; these and all the lectures were of the highest quality. Much was done in coffee breaks, over lunch, and in the evenings in never-ending discussions. Interdisciplinary dialogue was the goal of the symposium and as such, it was a great success.

The Symposium was organised by Ulrike Tillmann assisted by conference secretaries Brenda Willoughby and Laura Mildenhall. The Symposium was supported by the London Mathematical Society, the EPSRC, and Oxford University.

MARCO GUALTIERI

Talking with scientists

The application of their mathematical thinking and of the fundamental ideas of their subject to the problems of science, finance, engineering, and medicine is increasingly important to mathematicians. Problems as diverse as modelling the climate to modelling the growth of cancer cells are increasingly the province of the mathematician, where the ideas and techniques of stochastic analysis and partial differential equations are used to elucidate the physical phenomenon. The following sections describe how Professor Terry Lyons and his group use stochastic analysis to model the climate, how Professor Philip Maini's group apply partial differential equations to the modelling of tumour growth, and how Professor Jon Chapman uses singularities to model superconductivity.

Watching the weather

An undergraduate student studying the rigorous analysis of Weierstrass (1815–1897), the integration theory of Lebesgue (1875–1944), the mathematisation of probability by Kolmogorov (1903–1987), the martingales of Doob (1910–) and the stochastic differential equations of Itô(1915–) could be forgiven for believing that these subjects (each of which is, today, built on the predecessor) were so abstract that their only purpose was to provide an extremely interesting excuse for extending the undergraduate course from three to four years!

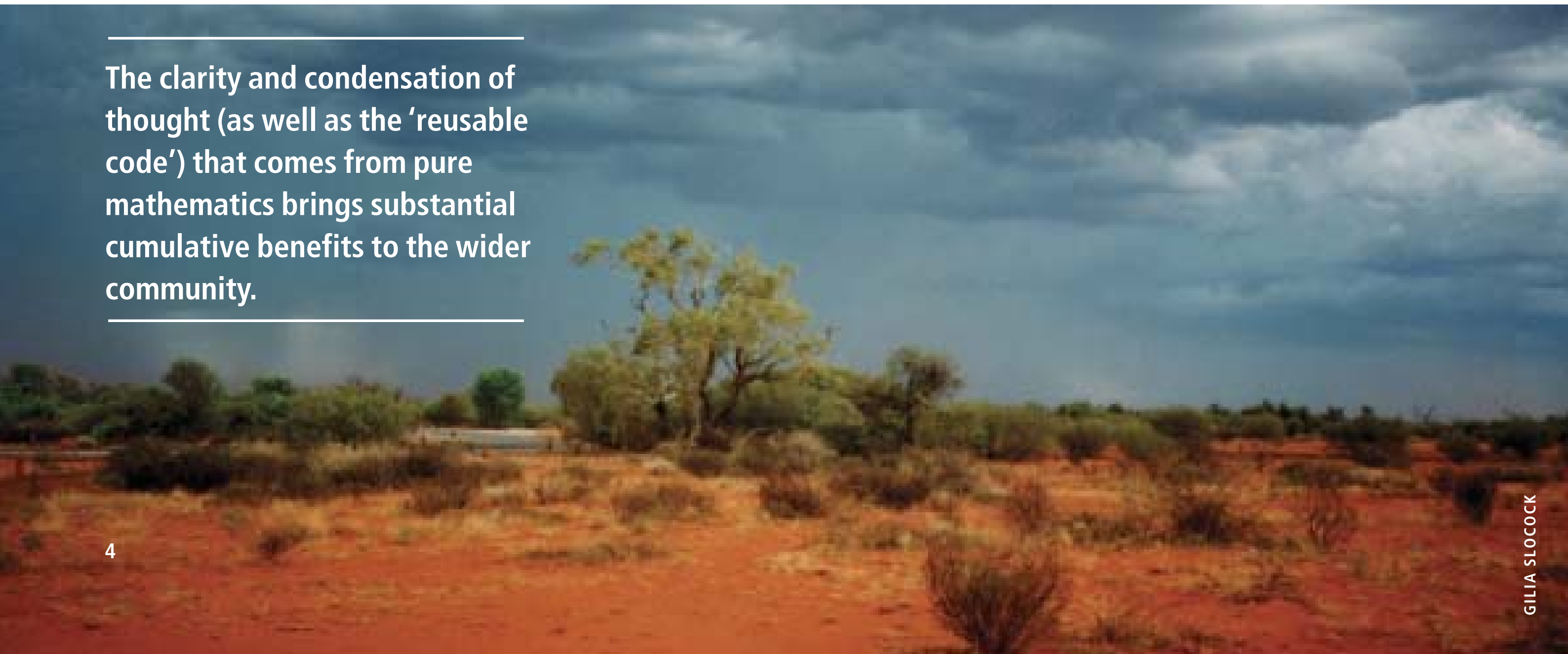
But the clarity and condensation of thought (as well as the 'reusable code') that comes out of pure mathematics brings substantial cumulative benefits to the wider community. Enabling a company to operate more efficiently by insuring it against currency and other price fluctuations may seem esoteric – but it reduces risk and, in competitive environments, allows tighter margins and lower prices. The trade in financial derivatives for this purpose is big business. The hedging that makes this possible involves the mathematics of Martingales and Stochastic Differential Equations, the core methodologies of stochastic analysis. A rather different use of ideas from stochastic analysis can be applied to modelling the climate.

Our climate is a good example of a complex system affected by random small-scale perturbations – for example, the effects of the localisation of precipitation, movements of the ocean surface, and even the geological features on the earth's surface. But there is an essential mathematical difficulty. On intermediate scales, where we hope to use the stochastic models, 'the stochastic microstructure' is not at all smoothly varying and the normal approach of coupling the external influences into the system through a differential equation needs refining: the controls are not differentiable.

Itô (with Itô calculus) used functional analysis to show that the classical Riemann approach could be pushed to give a rigorous way for almost every Brownian noise to control the evolution of a system, and using martingales this was extended to almost every path of a semi-martingale. The Itô calculus approach is particularly appropriate for the financial markets.

However, Itô's approach does not deal directly with other stochastic influences that fluctuate more wildly than Brownian motion. If we take a smooth control, and consider short time segments separately, then it is in the nature of smoothness that the controls coming from nearly adjacent time periods are essentially interchangeable. We can shuffle their order and the system will scarcely notice. In other words we can accurately summarise what happens over moderate time scales by associating with each time interval an element of a commutative group. We do this implicitly when we represent the control as a path in Euclidean space. But if the path has significant structure on the scale we are considering, then this approximation will not do; it turns out that it is better to summarise the path using an element of a non-commutative group. We then retain just enough information about the order of events on small scales to make predictions despite the fluctuations. The differential equations needed to model the evolution can be given precise meaning.

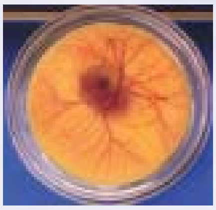
So now the fun begins – to feed these ideas into climate modelling and exploit the improved understanding of what we need to capture and model if we are to understand the influence of the random short time influences.



The clarity and condensation of thought (as well as the 'reusable code') that comes from pure mathematics brings substantial cumulative benefits to the wider community.

GILIA SLOCOCK

Investigating tumour growth



In vivo model of angiogenesis in chick embryo.

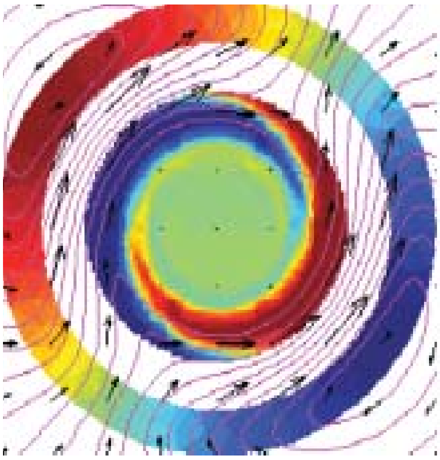
In the early stages of its development a solid tumour is no more than an isolated, harmless colony of cells limited in size by lack of nutrients. It needs a blood supply and access to the host's nutrients. Then at some stage the cells of the tumour begin to secrete certain substances called tumour angiogenic factors and these, on reaching the host's blood system, trigger the migration towards the tumour of endothelial cells which provide the tumour with its own blood supply. This process is called angiogenesis.

The tumour now has access to the host's nutrients and the means for metastasizing and forming secondary and usually fatal tumours. Targeting angiogenesis with drugs is seen as a possible way of combating cancer. However, before any drug can be developed — or tested – it is necessary to elucidate the process of cell growth, and this is where mathematical modelling comes in.

Research into these fundamental processes is the current focus of the Centre for Mathematical Biology which under Professor **Philip Maini** is the England team leader (in collaboration with the University of Nottingham) in a major research project funded by the EC and forming a network with centres in Scotland, Italy, Spain, Germany, Poland, Israel, and France. The focus of the research is to model the process of cancer mathematically from the molecular level to the organ level. These models are of coupled partial differential equation type set up to predict the spatiotemporal evolution of such key variables as tumour cell density, the angiogenesis factors, and the endothelial cell density.

A fundamental mathematical goal touching many different areas of science is the understanding and mathematical modelling of the large-scale systematic evolution of systems that are influenced by fine structure.

Superconductivity



Magnetic fields in a superconductor (G.J. Barnes, M.D. McCulloch, and D. Dew-Hughes).

One of the intriguing things about mathematics is the way in which equations that model one physical phenomenon can often be adapted to model another.

A superconductor is an element, inter-metallic alloy, or compound that will conduct electricity without resistance below a certain temperature. The key is to minimise resistance. However when the superconductor carries a current the interaction of the current with the magnetic field (the Lorentz force) causes resistance. The reason is as follows: the magnetic field inside a superconductor is typically arranged in a spaghetti-like network of thin 'flux tubes' which under the influence of current start to move about and dissipate energy which leads to an electrical resistance. It is therefore important to understand the motion of the flux tubes.

The mathematical modelling of the phenomenon has won Professor **Jon Chapman** the first Julian Cole Prize from the Society for Industrial and Applied Mathematics. One of the intriguing things about mathematics is the way in which equations that model one physical phenomenon can often be adapted to model another. For example the electrons in the superconductor swirl around the magnetic flux tube in exactly the same way as smoke swirls around in a smoke ring. Flux tubes can be modelled as singularities in the partial differential equations describing the superconducting material. Jon Chapman again took the analogy of vortices in fluids and was able to formulate a law of motion that determined the dynamics of the whole spaghetti-like structure of the flux tubes.

The business of problem solving

Pricing an option

No fewer than five Oxford graduate students will submit DPhil theses this year on mathematics applied to finance, while more than 60 will take a third-year course on option pricing, and a further 50 will come to Oxford to take a part-time Master's course. This is dramatic evidence of interest in a subject which almost seems to have come from nowhere, an academic discipline that scarcely existed a decade ago. The driving force has been the extraordinary growth in the use of high-level mathematics in banks and other financial institutions worldwide.

DAVID TOLLY

Pricing an option provides the prototype finance problem, the option being a contract that gives its holder the right, but not the obligation, to buy an asset such as a share for a specified price. This would be easy if the asset price evolved predictably but, of course, it does not. The solution of the problem involves the use of sophisticated ideas from probability and partial differential equations together with an ingenious strategy to reduce the risk in holding an option by trading in the asset. It earned Black, Scholes, and Merton the 1997 Nobel Prize in economics and, said Sam Howison, 'We were delighted to welcome Bob Merton to give the inaugural Nomura lecture at the launch of the Nomura Centre for Quantitative Finance'.

This centre, directed by Dr **Sam Howison** and funded by Nomura International plc, is one of two focal points for the mathematical finance group. The other is the Oxford Centre for Computational Finance, a joint venture with Physics and Computing; Dr **Jeff Dewynne** is one of its three directors. The Oxford Mathematical Finance Group, which is one of the largest in the UK, is pushing ahead on a variety of research topics as diverse as energy derivatives, Monte Carlo pricing for American options, stochastic volatility, agent-based complex systems models, behavioural finance, and detailed analysis of liquidity in order-book markets.

Above: Nobel Prize winner Professor Robert Merton (MIT) giving the inaugural Nomura lecture at the launch of the Nomura Centre for Quantitative Finance.

Right: Hilary Ockendon is Ex-Director of the Oxford Centre for Industrial and Applied Mathematics

Counting the cost

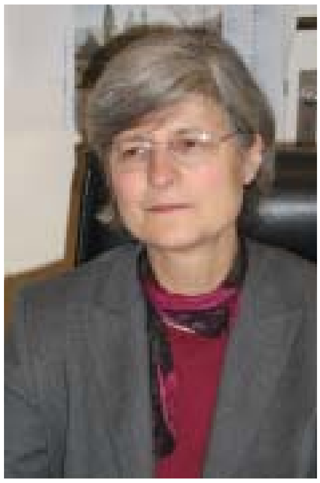
Oxford Combinatorics under Professor **Dominic Welsh**, is a major player in the consortium COMBSTRU or, to use its full title, Combinatorial Structure of Intractable Problems. Starting in October 2002, the programme has four-year EEC funding. The eight-country consortium includes members from the universities of Barcelona, Berlin, Budapest, and Prague.

Combinatorial structure lies at the core of many decision, optimisation and counting problems which are difficult to compute but are relevant in many scientific and technological areas. The objective is to build up a common framework for the analysis of intractable combinatorial problems using techniques from algebra, logic, geometry, probability, and statistical physics. The result will be the development of algorithms that can be applied to problems in telecommunications and biology.

Exchange visits, scientific meetings, and web-based information systems are planned to encourage communication and training for graduate students and researchers.

The opening workshop of COMBSTRU took place in Prague at the end of October 2002. Post- and pre-doctoral visitors will be in Oxford from January 2003.

OCIAM

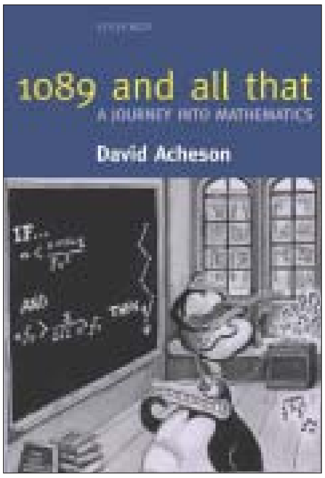


The Oxford Centre for Industrial and Applied Mathematics was created in 1989 as a centre, within the Mathematical Institute, which specialises in applying innovative mathematics to real problems in industry and applied science. The original focus of activity was the annual Study Group with Industry; this concept has proved an exemplary method of technology translation from mathematics to industry, which has now been adopted by groups of applied mathematicians all over the world. The European Study Group with Industry is now a collaborative project, which meets at least three times a year in various countries under the aegis of the European Consortium for Mathematics in Industry.

Since its inception the centre has nearly doubled in size and is now the largest research group in the Institute. The research emphasis is on continuum modelling of both deterministic and stochastic processes. There is still a large group, which includes many students, working on traditional industrial problems and associated mathematical methods but there are also groups working on finance, nonlinear prediction, medicine, and the environment. All the work is essentially collaborative and there are close research links with at least 10 other departments in Oxford as well as with many other research groups and industrial scientists both in the UK and abroad.

The world of books

A phenomenon of recent publishing has been the proliferation of general-interest books on cutting-edge aspects of science. Oxford mathematicians have seen great success in authorship and this coming year promises some interesting and provocative reading for scientist and layperson alike.



Sense of Wonder

David Acheson is well known for his textbook *Elementary Fluid Dynamics* but his most recent book is written for a 'totally-non-technical' readership, those readers for whom mathematics at school held little or no appeal.

What the critics say

'A lovely little book' Simon Singh, author of *Fermat's Last Theorem*.

'Anyone who is baffled by mathematics should buy it. And all mathematicians should buy at least a dozen copies to hand out to people they meet at parties. My enthusiasm for it knows no bounds.' Ian Stewart, author of *Does God Play Dice?*

Acheson, David *1089 and All That* Oxford University Press (2002) 0-19-851623-1

The Four-Colour Problem

Autumn 2002 marked the 150th anniversary of the celebrated four-colour problem – which asks, 'can every map be coloured with just four colours in such a way that neighbouring countries are coloured differently?' It also marks the twenty-fifth anniversary of the publication of its solution by Kenneth Appel and Wolfgang Haken of the University of Illinois. **Robin Wilson's** latest book, *Four Colours Suffice* celebrates these events.

Wilson, Robin *Four Colours Suffice* Allen Lane, The Penguin Press (2002) 0–713–99670–6

The Music of the Primes

The story of our attempts to unravel Riemann's great problem has taken mathematicians on a roller-coaster ride through the mathematical landscape. **Marcus du Sautoy** recounts the tale in his book *The Music of the Primes.*

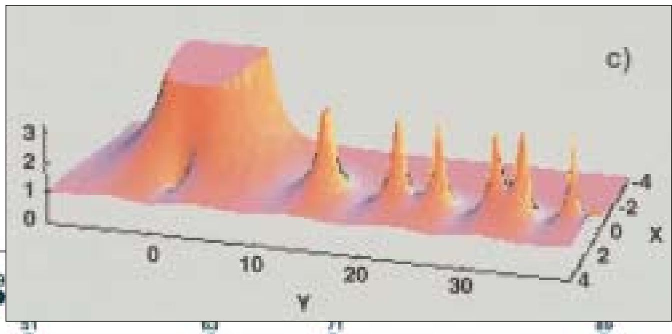
In 1900, David Hilbert of the University of Göttingen heralded the new century with a list of 23 problems that he believed should set the course for the mathematical explorers of the twentieth century. Only one of Hilbert's problems entered the twenty-first century virtually unscathed. This was the Riemann Hypothesis, a problem that goes back to the Greeks and has obsessed every generation since finding patterns in the Primes.

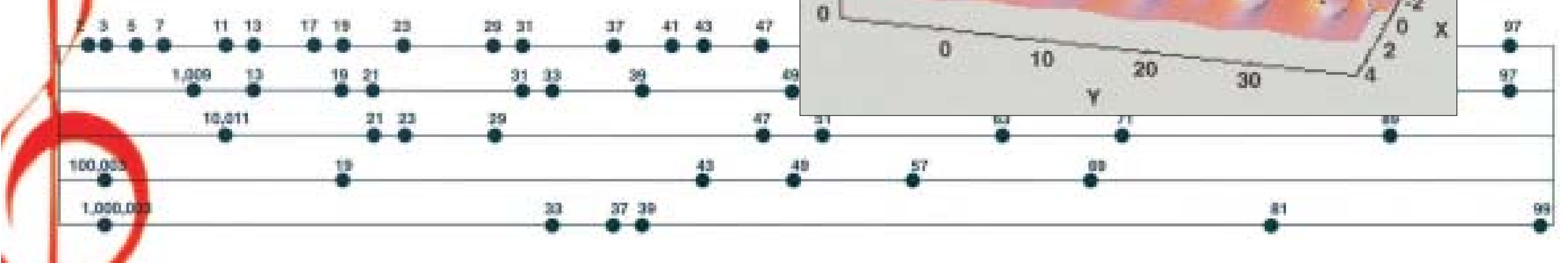
There is a critical reason why Riemann's Hypothesis and prime numbers are not the concern of mathematicians alone. Current state-of-the-art secret codes all rely on what is known and, more crucially, what is not yet known about prime numbers. Every moment of every day, through banking transactions, e-commerce on the Internet, and other electronic exchanges of confidential information, our lives are being touched and mostly protected by these codes.

Du Sautoy, Marcus *The Music of the Primes: the quest for the Riemann Hypothesis* Fourth Estate (2003) 1–84115–579–9

The Riemann Hypothesis (right): Riemann conjectured that all the peaks in this inverted image of the Riemann zeta function should lie in a straight line. If true, it will explain why the primes look so random.

The music of the primes (below). Primes in five 100-number intervals: 1-100, 1000-1100, 10000-10100, 100000-100100, 1000000-1000100, represented by five lines, with each prime shown as a dot.





Congratulations

Newcomers

The Institute welcomed the following new lecturers in October 2002

- **Dr Susan Howson**, New College, who brings with her from Nottingham her Royal Society Dorothy Hodgkin Fellowship.
- **Dr Bernd Kirchheim**, who joins Professsor John Ball's research group from the Max Planck Institute for Mathematics in Leipzig. He is a Fellow of Trinity College.
- **Dr Xenia de la Ossa**, who has been a postdoc here in string theory, now becomes a University Lecturer and Fellow of Oriel College.
- **Dr William Shaw** who has been with Nomura but also teaching at Balliol. He takes up a joint appointment with the Department for Continuing Education in Mathematical Finance, together with a tutorial fellowship at St. Catherine's.

Susan Howson

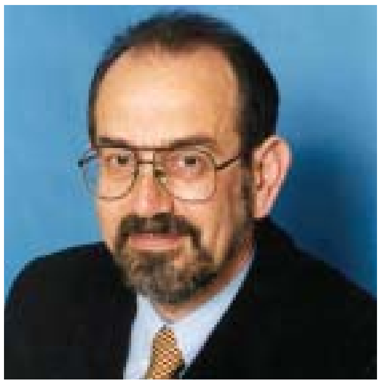
The Institute congratulates

- New professors: **Professor Marcus du Sautoy** and **Professor Dominic Joyce.**
- **Professor J.M. Ball** – from 1 January 2003 to be President of the International Mathematical Union.
- **Professor S.J. Chapman** – awarded the first Julian Cole Prize by the Society for Industrial and Applied Mathematics (SIAM).
- **Professor J.A. Green** – awarded the London Mathematical Society's De Morgan Medal at a special ceremony at the Institute in November. Sandy Green has been working in Oxford since his retirement.
- **Dr Susan Howson** – awarded the Adams Prize. Dr Howson's prize-winning work was on elliptic curves and cryptography. She is the first woman to receive this award.
- **Professor T.J. Lyons** – elected Fellow of the Royal Society. This brings to 14 the number of FRSs working at the Institute, one in six of the mathematical Fellows worldwide.

Shiing Shen Chern and John Ball at ICM 2002, Beijing.

The Polya Prize

The Polya Prize, one of the senior awards of the London Mathematical Society, is awarded to a member of the Society whose contribution to mathematics has been of high originality and influence. This year's winner is **Professor Nigel Hitchin**. He is the fourth Oxford mathematician in succession to win this prize: his immediate predecessors were John Hammersley (1997), Simon Donaldson (1999), and Terry Lyons (2000). Graeme Segal won the prize in 1990.



ROB JUDGES

Nigel Hitchin

The Mathematical Institute is a department of the University of Oxford

Website: **http://www.maths.ox.ac.uk**

Contacts

Head of Department, *Dr. N. M. J. Woodhouse,*
Mathematical Institute, 24–29 St Giles', Oxford OX1 3LB

Editor: Elizabeth Johnston. Design: Sarah Dickinson