Let $X$ be a Markov process taking values in $\mathcal{E}$ with continuous paths and transition function $(P_{s,t})$. Given a measure $\mu$ on $(\mathcal{E}, \mathcal{E}')$, a Markov bridge starting at $(s, \varepsilon_x)$ and ending at $(T^*, \mu)$ for $T^* < \infty$ has the law of the original process starting at $x$ at time $s$ and conditioned to have law $\mu$ at time $T^*$. Two types of conditioning will be considered: a) weak conditioning when $\mu$ is absolutely continuous with respect to $P_{s,t}(x, \cdot)$ and b) strong conditioning when $\mu = \varepsilon_z$ for some $z \in \mathcal{E}$. The representation of a Markov bridge as a solution to a stochastic differential equation (SDE) driven by a Brownian motion in a diffusion setting will be obtained. Under mild conditions on the transition density of the underlying diffusion process the existence and uniqueness of weak and strong solutions of this SDE will be established.