**Subject panel:** Numerical Analysis  
**Suggested title of dissertation:** $C^\infty$ but nowhere analytic functions  
**Dissertation supervisor:** Nick Trefethen  
**Suggested second assessor:** Endre Süli

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**Description of proposal**

A central topic of both theoretical and numerical analysis is regularity. An appealing window into this very large subject is the question, what functions are there that are $C^\infty$ but nowhere analytic? One way to construct such a “smoothie” is via a Fourier series with random coefficients with root-exponentially decaying amplitudes.

**Possible avenues of investigation**

The dissertation may or may not be numerical. It will need to review some theory, and then there are many potential directions for further investigation including (1) deterministic vs. stochastic constructions, (2) links with lacunary series, (3) smoothies as boundary values of analytic functions in the unit disk, (4) ODEs based on smoothies, and (5) multidimensional smoothies.

**Pre-requisite knowledge**

Essential: complex analysis, real analysis. Useful: stochastic analysis.

**Useful reading**


**Further references**


Wikipedia article on “Non-analytic smooth function”.