J is for juggling

Juggling is the act of iteratively catching and throwing several objects. A juggling pattern can be described using a mathematical notation called siteswap. The idea of siteswap notation is to keep track of the order in which the objects are thrown.

A juggling pattern is a sequence of beats – each beat is a moment in time at which the juggler catches an object, and then immediately throws it. Each throw is represented by a positive number, the number of beats until this object needs to be caught (and rethrown). The siteswap notation for a juggling trick is a sequence of whole numbers representing the heights of the throws occurring at each beat in order.

When an even number is thrown, it will be caught by the same hand that threw it, whereas an odd-numbered throw will be caught by the opposite hand.

Siteswap notation is an effective way to describe juggling patterns. It is also a useful tool for determining whether an arbitrary sequence is "juggleable" or not. For example, no siteswap sequence can contain a 4 immediately followed by a 3, because it is considered impossible to catch two balls arriving at the same hand at the same time.

The above shows a 3-ball cascade. Each term in the sequence is thrown alternately by the left and right hands.

There is another popular juggling pattern called the "shower", where the right hand always throws high (a 5 in siteswap notation) while the left hand always throws a quick horizontal pass (a 1 in siteswap notation) to the right hand.

Siteswap notation has even led to the discovery of previously unknown juggling patterns, for example the 4413 (above). Juggling has inspired mathematics, and mathematics has inspired juggling!

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For more about juggling visit www.maths.ox.ac.uk/r/alphabet