Modelling the carding of recycled carbon fibre

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1. The Carding Machine

- The process of recycling carbon fibre involves recovering waste carbon fibre and turning it into non-woven materials that can be used in industry.
- Carding machines consist of a number of rollers rotating at different velocities and in different directions.
- An unordered web of carbon fibres is fed into the machine, see figure 2.
- The fibres are then aligned as they move through the carding machine, forming a homogeneous, oriented web, see figure 3.
- The three main rollers we focus on are the swift (or main cylinder), the worker, and the stripper.
- Figure 1 shows the typical path of a fibre (order shown by numbers (1)-(7)) through this part of the carding machine.

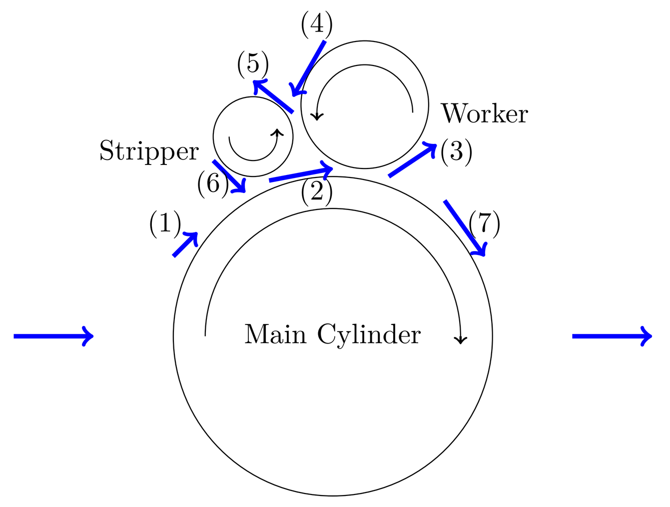

Figure 1 – Typical route of a fibre (shown in blue) through the carding machine.

Figure 2 – A photo of a web of carbon fibres before carding.

Figure 3 – A photo of a nonwoven of carbon fibres after carding and needle punching.

2. Challenge & Aims

Challenges:
- Produce a continuum model for the flow of material through the carding machine.
- Predicting how the control parameters affect the carding process.

Aims:
- A validated model for the whole carding machine.
- A mathematical tool for optimising the quality of the product.

3. The Model

- Adapt an existing model from the textile industry [1].
- Focus on the region between the worker and main cylinder, see figure 4.
- Define an order parameter that is equal to 1 when fibres are fully ordered, and 0 when they are randomly aligned.
- Model the order of the material increasing due to it being stretched.
- This region is very thin so we can make a lubrication-type approximation in 2D. Also approximate the surface of the larger cylinder as flat.

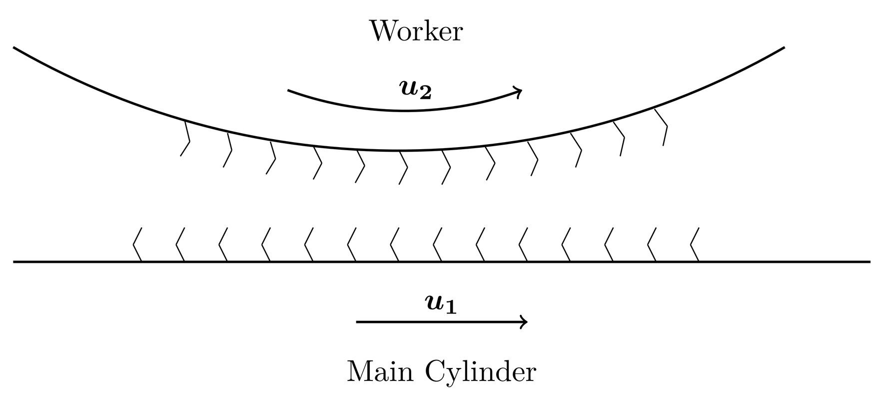

Figure 4 – Schematic of the region between the worker, moving at velocity u_2, and the main cylinder, moving at velocity u_1. The hooks along the surface of the cylinders, and their relative orientation, are also shown.

- Assume that the fibres are naturally straight and can therefore not entangle with each other, a significant difference to fibres in the textile industry.
- We derive an anisotropic, continuum model.
- The model is viscous, so we have the rate-of-strain tensor, e_{ij}, and no elasticity. It is similar to some anisotropic fluid and liquid crystal models, with the important differences of being compressible and there being no pressure.
- Important dependent variables are density of fibres, ρ, director, $\boldsymbol{a} = (\cos\theta, \sin\theta)$, where θ is the mean angle the fibres make with the horizontal, the order parameter, ϕ, and the velocity, (u, v).
- The model consists of equations for conservation of mass, conservation of momentum, a kinematic condition, and a constitutive law for the order parameter.

4. Governing Equations

Conservation of Mass

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0,$$

Conservation of Momentum

$$F_i + \frac{\partial\sigma_{ik}}{\partial x_k} = 0,$$

Constitutive relation for the stress including isotropic response, σ^I, and non-isotropic response, σ^N. Assume stress tensor linearly proportional to the rate-of-strain and the number of fibre contact points (the square of the density).

$$\begin{aligned}
\sigma_{\alpha\beta} &= (1-\phi)\sigma^I_{\alpha\beta} + \phi\sigma^N_{\alpha\beta}, \\
\sigma^I_{\alpha\beta} &= \rho^2\left(\mu_1 e_{\alpha\beta} + \mu_2 e_{kk}\delta_{\alpha\beta}\right), \\
\sigma^N_{\alpha\beta} &= \rho^2\Big(\lambda_1 e_{\alpha\beta} + \lambda_2 e_{kk}\delta_{\alpha\beta} + \lambda_3(a_\alpha a_i e_{\beta i} + a_\beta a_i e_{i\alpha}) \\
&\qquad + \lambda_4(a_\alpha a_\beta e_{kk} + \delta_{\alpha\beta} a_k a_l e_{kl}) + \lambda_5 a_\alpha a_\beta a_k a_i e_{kl}\Big),
\end{aligned}$$

Kinematic condition for director $\boldsymbol{a}$ convected by velocity $\boldsymbol{u}$.

$$\frac{\partial a_i}{\partial t} + u_k\frac{\partial a_i}{\partial x_k} + a_k a_l \frac{\partial u_k}{\partial x_l} a_i = a_k \frac{\partial u_i}{\partial x_k},$$

Constitutive law for order parameter - Assume evolution of order is proportional to the rate-of-extension but does not decrease under compression.

$$\frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = \left[\beta(1-\phi)a_k a_l \frac{\partial u_k}{\partial x_l}\right]^+.$$

5. Results

- Assuming a thin region between the cylinders leads to a lubrication-type approximation.
- To simplify, assume uniform viscosity and no time dependence. We can solve the momentum equations for u and v and use these to calculate the other variables.
- Assumptions allow progress to be made analytically so we can view the evolution of the order and density of fibres.

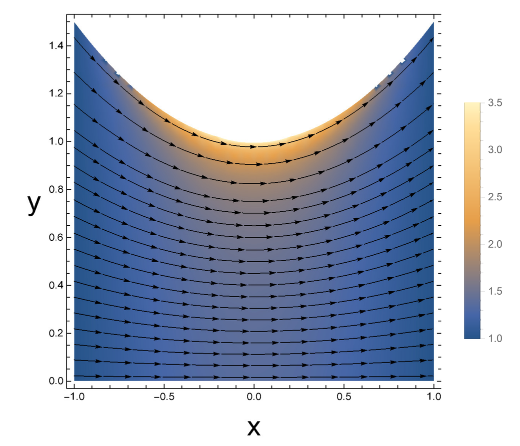

Figure 5 – Distribution of the density of fibres.

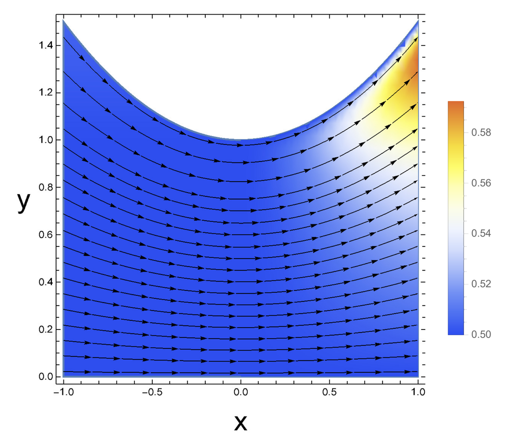

Figure 6 – Distribution of the order of fibres.

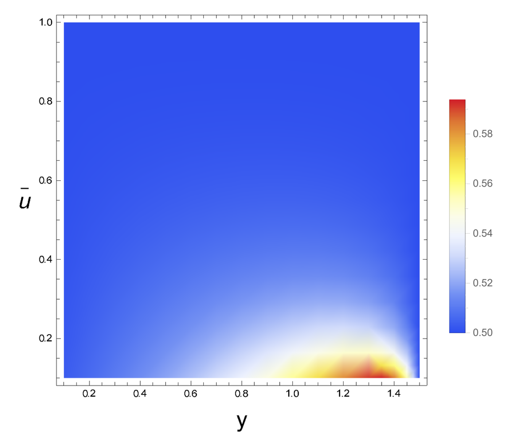

Figure 7 – Plot of order parameter at downstream end point.

- Density increases as web is compressed.
- Highest density near worker.
- Decreases again as web expands.
- The streamlines are shown in black.

- Order parameter increases across the region between the worker and main cylinder.
- Order only starts to change when the web is being stretched.
- Highest order occurs at an interior point near the worker.

- Order parameter at the downstream end point versus velocity ratio between the cylinders, $\bar{u}$.
- A large difference in the velocities of the cylinders is desirable.

6. A Combing Model

- Previously we assumed that the order of the fibres increases due to the material being stretched, and we used a continuum approach to modelling the web of fibres.
- In addition, there are hooks along the cylinders (as shown in figure 4), which increase the order by combing the material.
- To understand the effects of combing, we consider a simple model problem in which a hook moves through a web of three crossed fibres.

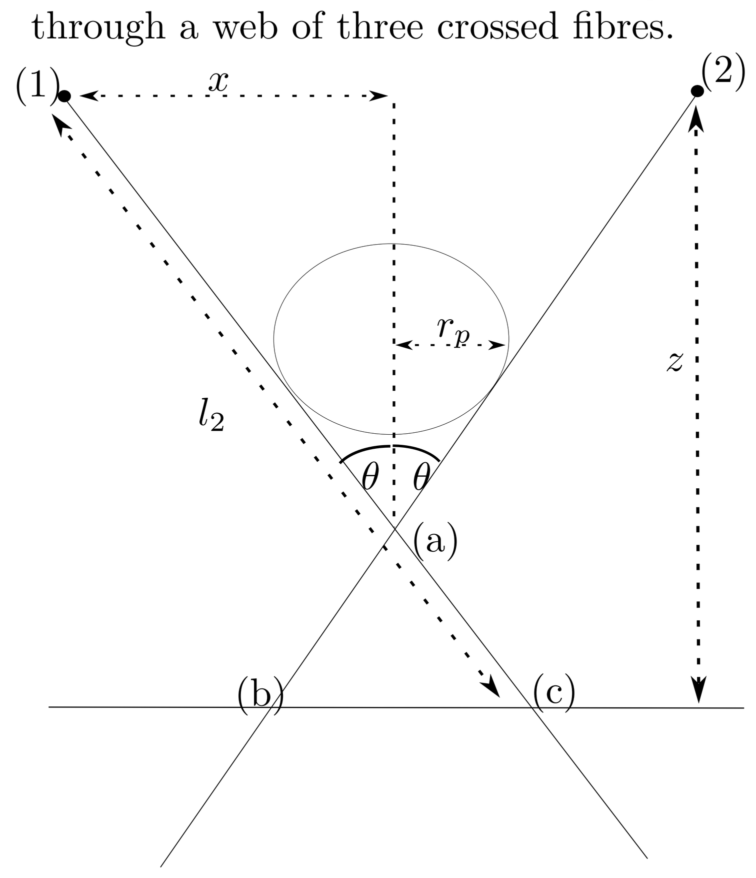

Figure 8 – Schematic of a pin being dragged downwards to comb three overlapping fibres.

- We consider a hook of radius r_p moving at constant velocity, V, through 3 fibres.
- We assume that the top 2 fibres are fixed at the end points (1) and (2), due to the presence of other fibres in the surrounding material.
- The hook is dragged downwards with force F.
- The angle θ decreases as the fibres are combed until the hook can pass through.
- The fibres are modelled as straight rods.
- We want to know the force, F, required to comb at a constant velocity without the tension in the fibres, T, becoming large enough for the fibres to break.
- We use Coulomb friction to describe the interaction between the hook and the fibres, with coefficient of friction μ.
- We assume that the frictional force is a function of the relative velocity of the fibres at crossover points (a), (b), and (c).

Resulting Equations:

$$F = \frac{AV\cos^2\theta\sin\theta\left(x + \frac{l_2^2}{x}\sin^2\theta\right)}{x\left(1 - \frac{r_p}{x}\cos\theta\right)^2}(\mu\cos\theta + \sin\theta), \quad T = (x\sin\theta + l_2)\frac{AV\cos\theta\sin\theta}{x\left(1 - \frac{r_p}{x}\cos\theta\right)}$$

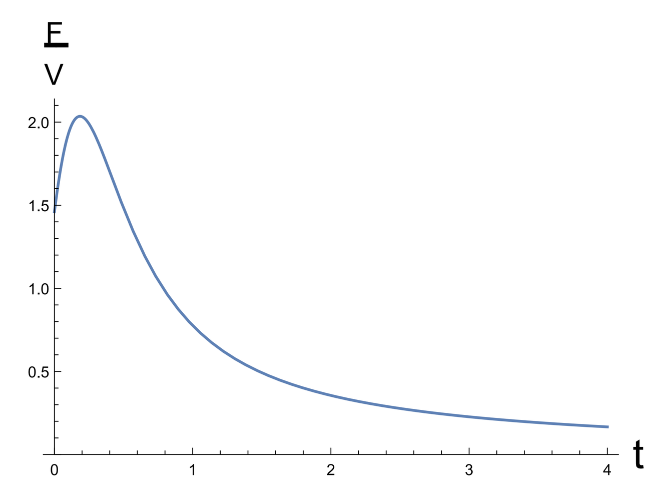

Figure 9 – The force required to move the hook through the fibres at a constant speed against time.

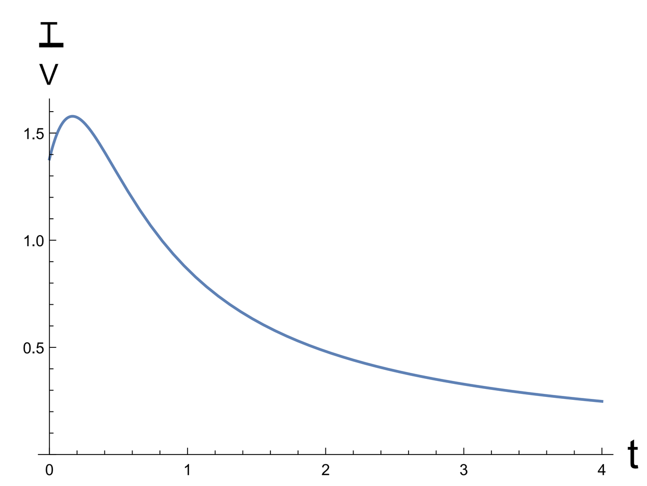

Figure 10 – The tension, T, in the fibre as the hook is moved through the fibres at a constant speed against time.

7. Conclusions & Further Work

- We have derived a continuum model for fibres moving through a carding machine, in a lubrication-type approximation.
- Our model shows the order of the fibres increasing in the region between the main cylinder and the worker, agreeing with observations.
- A higher velocity difference results in more efficient carding.
- Started developing a microscale model to describe combing.
- We will incorporate the effect of the hooks in the macroscale by adding point forces to our momentum equations.

Acknowledgements

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ELG Carbon Fibre Ltd. RECYCLED CARBON FIBRE

References

[1] M. E. M. Lee and H. Ockendon. A continuum model for entangled fibres. *European Journal of Applied Mathematics*, 16(2):145–160, 2005.