12th Oxford-Berlin Young Researchers Meeting on Applied Stochastic Analysis

Mathematical Institute, University of Oxford

4^{th} December – 6^{th} December 2019

Supported by DATASIG EP/S026347/1

Contents

1. Welcome to Oxford Mathematics

On behalf of the Mathematical Institute it is our great pleasure to welcome you to the University of Oxford for the **12th Oxford-Berlin Young Researchers Meeting on Applied Stochastic Analysis**. We hope you enjoy a productive meeting in Oxford.

Conference organisers
Terry Lyons, Peter Friz, Patric Bonnier, Carlo Bellingeri

This meeting is generously supported by the European Research Council under the DATASIG grant.

Venue

All talks take place in the Mathematical institute in the following lecture theatres:

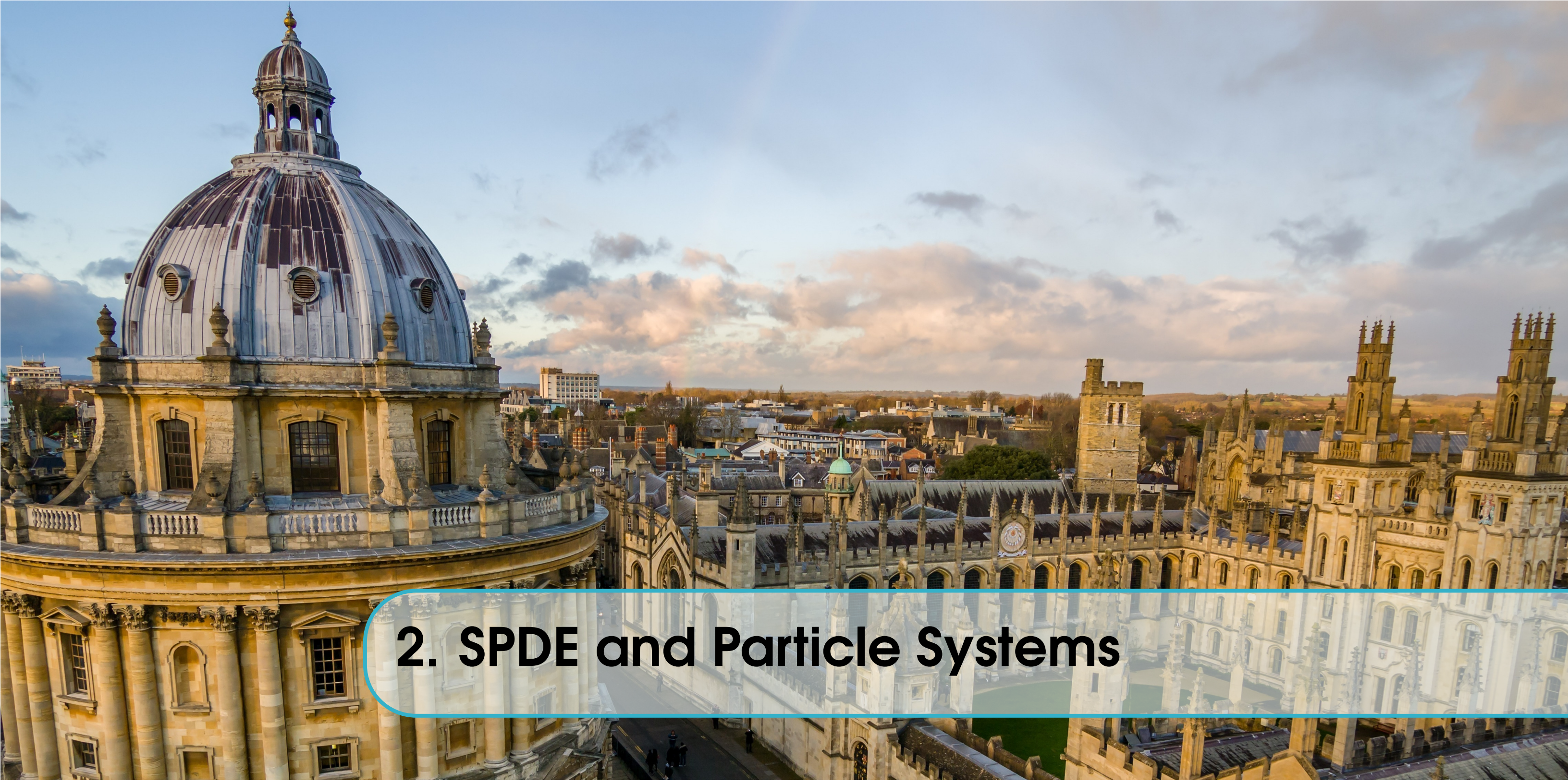
Wednesday December 4th	L1	Basement
Thursday December 5th	N4.01	Top floor of North Wing
Friday December 6th	C2	Basement

Presentations

All talks are 25 minutes long including 5 minutes for questions. The lecture rooms are equipped with a computer, projector and whiteboards. Please send in the slides for your talk by 9am of the day you are presenting.

Refreshments

Coffee/tea will be provided.



2. SPDE and Particle Systems

2.0.1 Large deviations in interacting particle systems and stochastic PDE, *Benjamin Fehrman, Oxford*

In this talk, which is based on joint work with Benjamin Gess, we will draw the link between large deviations in interacting particle systems and large deviations for certain classes of stochastic PDE. The motivating example will be the large deviations of the zero range process about its hydrodynamic limit. We will show informally that the corresponding rate function is identical to the rate function appearing for a degenerate stochastic PDE with nonlinear, conservative noise. Our primary result is a rigorous proof of this fact based upon an intricate treatment of the corresponding skeleton PDE, which is a nonlinear, energy-critical advection-diffusion equation.

2.0.2 From weakly interacting particles to a regularised Dean-Kawasaki model, *Federico Cornalba, IST Austria*

The Dean-Kawasaki (DK) model is a mass-conserving, parabolic SPDE describing mesoscopic fluctuations of a Langevin particle system in a linear mobility regime. This equation features a deterministic gradient-flow-driven drift, as well as a non-Lipschitz noise term in spatial divergence form. The DK model is used and simulated in physics, e.g., in the description of thin liquid films, of nucleation for colloids and macromolecules, and of nonequilibrium bacterial dynamics. However, very little is known in terms of analytical and numerical well-posedness. This is mainly due to the noise irregularity, which can be blamed on its conservative form and its lack of Lipschitz properties. We derive a regularised version of the DK model, based on a system of weakly interacting particles following underdamped McKean-Vlasov dynamics. We provide high-probability existence and uniqueness results for our regularised model, and detail relevant open questions. This is joint work with Tony Shardlow and Johannes Zimmer.

2.0.3 A coupling approach for the rate of convergence of a collisionless gas, *Armand Bernou, Sorbonne Université*

We present a coupling approach for the rate of convergence towards equilibrium of a free-transport equation with boundary conditions, used to model the evolution of a collisionless gas. We will first define a stochastic process and sketch how to link this process with the equation. We will then construct two correlated processes that will be equal for ever starting from some random time. The moments of this random time will lead us to the rate of convergence predicted numerically by Tsuji, Aoki and Golse in 2010. One of the key difficulty is to make this coupling Markovian, as the randomness only occurs when one of the process hits the boundary of the domain. This is joint work with N. Fournier (SU).

2.0.4 Monotonicity of the Poincaré and Log-Sobolev Constants of Gibbs Measures, *Julian Sieber, Imperial*

Let $d\mu_\beta \propto e^{-\beta V(x)}dx$ be a Gibbs probability measure on a symmetric, compact interval I and assume that V is an even function. The measure μ_β is well known to satisfy both a Poincaré and Log-Sobolev inequality. We shall denote the sharp constants in these inequalities by $\alpha_\beta^{\mathrm{P}}$ and $\alpha_\beta^{\mathrm{LSI}}$, respectively. The Bakry-Émery criterion implies that, if $V''(x) \geq \kappa > 0$ for all $x \in I$,

$$\alpha_\beta^\sharp \leqslant \frac{1}{\kappa\beta}, \qquad \sharp \in \{\mathrm{P}, \mathrm{LSI}\}.$$

Motivated by this bound, we investigate the monotonicity of the mapping $\beta \mapsto \alpha_\beta^\sharp$. For the Poincaré constant, our result shows that this question can be answered from the monotonicity of V. This holds also true for non-convex V. We shall further illustrate by an easy example that there is no hope for a similar result for the Log-Sobolev constant.

2.0.5 Stochastic McKean-Vlasov Equations: Local and Global Theory in Dimension One., *Avi Mayorcas, Oxford*

We discuss the global well-posedness for a family of parabolic McKean–Vlasov SPDEs with additive space-time white noise. The family of interactions we consider are those given by convolution with kernels that are at least integrable. We show that global well-posedness holds in both the repulsive/defocussing and attractive/focussing cases conditional on uniform bounds on the density of the solution. Our strategy relies on both pathwise and probabilistic techniques which leverage the Gaussian structure of the noise and well known properties of deterministic PDEs.

2.0.6 LLN for particle systems via an SPDE in mild formulation, *Florian Bechtold, Sorbonne Université*

Consider a system of interacting particles perturbed by Brownian motion. While it is classical knowledge that the associated empirical measure converges to the solution of a McKean-Vlasov PDE (which can be seen as a law of large numbers), typical proofs of this are quite technical, involving e.g. a tightness argument. We present an alternative proof by showing that the empirical measure associated to particle systems of fixed size n satisfies (in the mild sense) a stochastic partial differential equation with additive noise, which can be shown to converge to the corresponding deterministic McKean-Vlasov PDE as $n \to \infty$. This is joint work in progress with Fabio Coppini (LPSM, Université de Paris).

2.0.7 Scaling limits and homogenization of Hamilton-Jacobi equations with stochastic forcing, *Benjamin Seeger, UPD CEREMADE*

We present results concerning the stochastic homogenization of Hamilton-Jacobi equations forced by stochastic and/or mixing temporal noise. Emphasis will be placed on the wide variety of possible limiting problems that arise, depending on the scaling regime and the structure of the equations. In particular, the effective equation can be stochastic or deterministic, with the noise having an enhancement effect in the latter case, or explosion can occur. We also discuss some new regularity results, which are an important tool in some of the proofs and are of independent interest.

3. Data Science and Signatures

3.0.1 Deep Signature Transforms, *Patrick Kidger, Oxford*

The signature transform has previously been treated as a fixed feature transformation, on top of which a model may be built. The talk will decribe a general approach which combines the advantages of the signature transform with modern deep learning frameworks. In particular, by learning an augmentation of the stream prior to the signature transform, the terms of the signature may be selected in a data-dependent way. More generally, we describe how the signature transform may be used as a layer anywhere within a neural network. In this context it may be interpreted as a pooling operation.

3.0.2 Learning SDEs using RNN with Log-Signature Features, *Shujian Liao, UCL*

Learning a function of streamed data through evaluation is an important question in many scientific areas. The difficulty comes from the high frequency of the driven stream. Motivated by the numerical approximation theory of the stochastic differential equations, I will introduce a novel approach (Logsig-RNN) combining the log-signature as representations of streamed data and the recurrent neural network (RNN). The ability of the former to manage high-frequency streams and the latter to manage large scale nonlinear interactions allows hybrid algorithms that achieve higher accuracy, are quicker to train. By testing on various datasets, i.e. synthetic data, NTU RGB+D 120 action data and Chalearn 2013 gesture data, I will show our proposed approach achieved state-of-art accuracy with high efficiency and robustness.

3.0.3 Learning with signatures: embeddings and truncation order selection, *Adeline Fermanian, Sorbonne Université*

Sequential and temporal data arise in many fields of research, such as quantitative finance, medicine, or computer vision. We will be concerned with a novel approach for sequential learning, called the signature method, and rooted in rough path theory. Its basic principle is to represent multidimensional paths by a graded feature set of their iterated integrals, called the signature. On the one hand, this approach relies critically on an embedding principle, which consists in representing discretely sampled data as paths, i.e., functions from $[0,1]$ to R^d. We investigate the influence of embeddings on prediction accuracy with an in-depth study of three recent and challenging datasets. We show that a specific embedding, called lead-lag, is systematically better, whatever the dataset or algorithm used. On the other hand, in order to combine signatures with machine learning algorithm, it is necessary to truncate these infinite series. Therefore, we define an estimator of the truncation order and prove its convergence in a signature regression model.

3.0.4 Variational Gaussian processes with signature covariances, *Csaba Toth, Oxford*

The space of (geometric or branched rough) paths can be equipped with a natural inner product using the signature lift. This so-called signature kernel can be used as a covariance function to define a real-valued Gaussian process indexed by paths. Besides being an interesting mathematical object, it connects ideas from stochastic analysis with Bayesian inference. As a concrete application, we develop a variational approach to learn the posterior of this process indexed by paths.

4. Algebraic Aspects of Signatures

4.0.1 Signature Cumulants and Ordered Partitions, *Patric Bonnier, Oxford*

The sequence of so-called signature moments describes the laws of many stochastic processes in analogy with how the sequence of moments describes the laws of vector-valued random variables. However, even for vector-valued random variables, the sequence of cumulants is much better suited for many tasks than the sequence of moments. This motivates the study of so-called signature cumulants. To do so, an elementary combinatorial approach is developed and used to show that in the same way that cumulants relate to the lattice of partitions, signature cumulants relate to the lattice of so-called "ordered partitions". This is used to give a new characterisation of independence of multivariate stochastic processes.

4.0.2 Signatures of paths transformed by polynomial maps, *Rosa Preiß, TU Berlin*

Joint work with Laura Colmenarejo

We characterize the signature paths transformed by a polynomial map in terms of the signature of the original path. For this aim, we define recursively an algebra homomorphism between two shuffle algebras on words. This homomorphism does not depend on the path and behaves well with respect to composition and homogeneous maps. We furthermore give a generalisation in terms of homomorphisms of Zinbiel algebras and some first ideas on applications in stochastic analysis.

4.0.3 Iterated-sums signature, quasi-symmetric functions and time series analysis, *Nikolas Tapia, WIAS/TU Berlin*

We survey and extend results on a recently defined character on the quasi-shuffle algebra. This character, termed iterated-sums signature, appears in the context of time series analysis and originates from a problem in dynamic time warping. Algebraically, it relates to (multidimensional) quasi-symmetric functions as well as (deformed) quasi-shuffle algebras.

5. Singular SPDE

5.0.1 Boundary renormalisation of stochastic PDEs, *Mate Gerencse, IST Austria*

We discuss solution theories of singular SPDEs endowed with various boundary conditions. In several examples nontrivial boundary effects arise and another layer of renormalisation is required. We outline how these are connected to spatial singularities of simple trees in equations like the KPZ, PAM, of Φ^4.

5.0.2 A Low Temperature Peierls Theory for Φ^4_3, *Trishen Gunaratnam, University of Bath/Imperial*

Peierls' argument for the 2D Ising model is one of the most elegant proofs of phase transition in statistical mechanics. This was famously extended to the Phi^4_2 Euclidean field theory by Glimm, Jaffe and Spencer in the mid '70s. Their proof fails for Φ^4_3 due to significant difficulties coming from the renormalisation theory, although a softer method exists to show the phase transition.

In this talk, we show how one can develop a Peierls-type theory for low temperature Φ^4_3 using the continuous renormalisation group approach of Barashkov and Gubinelli. Unlike in the 2D case, large scales and small scales need to be treated separately. The small scales (cancellation of ultraviolet divergences) are treated as in Barashkov and Gubinelli. The large scales are treated by an argument inspired by Glimm, Jaffe and Spencer. Making the argument non-perturbative requires energy estimates with a non-convex potential along with chessboard estimates.

We highlight two applications:

1. A quantitative approach to phase coexistence and separation for Φ^4_3;
2. A rate of decay of spectral gap for the low temperature, dynamical Φ^4_3 model in the thermodynamic limit.

Joint work with Ajay Chandra and Hendrik Weber

5.0.3 Global solutions for quasilinear SPDEs via the boundedness-by-entropy method, *Alexandra Neamtu, TU Münich*

We investigate the well-posedness of quasilinear SPDEs, where the diffusion matrix is neither symmetric nor positive-semidefinite. Such problems arise in many application areas like fluid dynamics, cell biology and biofilm modeling. We explore a formal gradient-low or entropy structure of these equations and an interplay between the entropy density and the stochastic terms in order to investigate properties of the solution. This talk is based on a joint work with G. Dhariwal, F. Huber, A. Jüngel (Vienna University of Technology) and C. Kuehn (Technical University of Munich).

5.0.4 A variational approach to Φ^4_3, *Nikolay Barashkov, IAM/HCM University of Bonn*

I will present a variational construction of the Φ^4_3 measure on a periodic domain well known from constructive field theory. In this approach one uses a stochastic control representa- tion of the Laplace transform of a regularized measure. Then using Ito's formula along a scale parameter and the paracontrolled calculus introduced first in the context of singular SPDE's by Gubinelli, Imkeller and Perkowski one extends this characterization to the non-regularized model. As a byproduct one can also represent Φ_3^4 as an absolutely continuous perturbation of a measure obtained from the Gaussian free field by Girsanov transform, which also gives a proof of singularity with respect to the free field. This is joint work with M.Gubinelli.

5.0.5 Strichartz estimates for the Anderson Hamiltonian, *Immanuel Zachhuber, Bonn University*

After recalling some recent developments related to the continuum Anderson Hamiltonian in 2- and 3-dimensions I will present some new results that combine tools from singular SPDEs, namely Paracontrolled Distributions, with a method due to Burq/Gerard/Tzvetkov. This gives rise to Strichartz-type estimates which allow to solve multiplicative stochastic NLS in a low-regularity regime.

5.0.6 From periodic to Dirichlet and Neumann on boxes, *Willem van Zuijlen, WIAS Berlin*

In this talk I will discuss Dirichlet and Neumann Besov spaces on boxes. The Bony estimates for para- and resonance products extend to these spaces. This is part of the work with Khalil Chouk that allows us to define the Anderson Hamiltonian with Dirichlet (and Neumann) boundary conditions by using the construction of the operator on the torus, i.e., with periodic boundary conditions.

TBC, *Harprit Singh, Imperial*

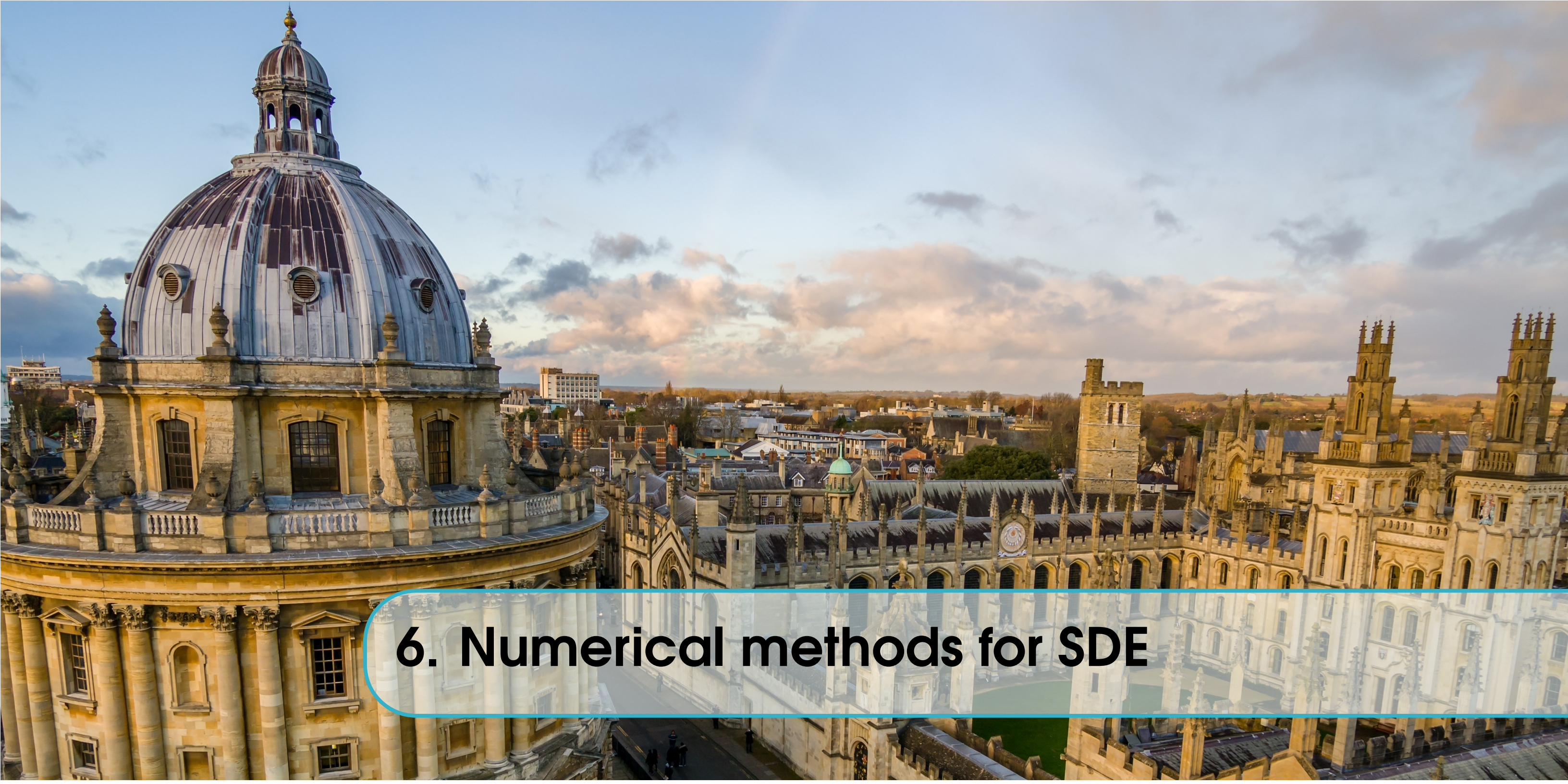
TBC

5.0.7 Multidimensional SDE with distributional drift and Lévy noise, *Helena Kremp, FU Berlin*

We solve multidimensional SDEs with distributional drift of rather general regularity driven by a symmetric, α-stable Lévy process for $\alpha \in (1,2]$. To this end, we introduce the notion of a (singular) martingale problem and solve the backward generator PDE in the case when the Besov regularity of the drift is below the critical threshold for the classical PDE theory of $(1-\alpha)/2$. Therefore, we employ the theory of paracontrolled distributions estabished by Gubinelli, Imkeller and Perkowski to solve singular PDEs. We prove existence and uniqueness of a solution to the martingale problem associated to the Lévy SDE with Besov drift. Our work extends a recent result of Cannizzaro and Chouk from the Brownian to the stable Lévy noise case. If time permits, we apply our theory to construct the Brox diffusion with Lévy noise $dXt = \xi(Xt)dt + dLt, X0 = x \in R$, where ξ is periodic white noise on R independent of L.

5.0.8 Random directed mean curvature flow, *Andris Gerasimovics, Imperial*

We consider a directed mean curvature flow in 2 dimensions, perturbed by a smooth and stationary noise. The dynamics of the curve undergoing the above flow can be described with a quasi - linear SPDE. Using the techniques of regularity structures we show that under a certain rescaling solution of such SPDE converges to the solution of the KPZ equation. In particular we show how to rewrite this SPDE as a system of subcritical equations which guarantees the convergence of the BPHZ model. The same trick can be also applied to put the Hairer-Quastel result immediately into the framework of the black box by Bruned, Chandra, Chevyrev and Hairer. Joint work with Martin Hairer and Konstantin Matetski.



6. Numerical methods for SDE

6.0.1 High order numerical simulation of the Cox-Ingersoll-Ross process, *James Foster, Oxford*

The Cox-Ingersoll-Ross (or CIR) model is a classic example of a scalar SDE that is difficult to accurately simulate. The main challenge is that the square root vector field is not globally Lipschitz and has derivatives that "blow up" near zero. To complicate matters further, the process (and thus its approximation) cannot be negative.

In this talk, we will present a new discretization of the CIR process based on two key ideas.

Firstly, by replacing the driving Brownian motion with a piecewise linear path, it is possible to use an implicit Runge-Kutta method to compute a numerical solution. Due to the analytically tractable vector fields, this approach is efficient and guarantees positivity.

Secondly, the above method can be incorporated into a variable step size framework. By taking smaller steps near zero, we can control the approximation error due to the large vector field derivatives.

We shall conclude the talk with numerical evidence that suggests this methodology exhibits high orders of both strong and weak convergence.

6.0.2 Numerical methods for SDEs - a stochastic sewing approach, *Oleg Butkovsky, WIAS*

(joint work with Konstantinos Dareiotis and Máté Gerencsér) We give a new take on the error analysis of approximations of SDEs, utilizing the stochastic sewing lemma (SSL) of Le. This approach allows to exploit regularization by noise effects in obtaining convergence rates. We establish rate of convergence of the Euler-Maruyama scheme for SDEs driven by fractional Brownian motion with non-regular drift. One of our key tools is a new version of the stochastic sewing lemma which complements SSL of Le.

7. Rough Paths

7.0.1 Renormalisation and the Hairer-Kelly map, *Yvain Bruned, Edinburgh*

The Hairer-Kelly map shows that a Branched Rough Path can be encoded as a Geometric Rough Path. A recent renormalisation procedure given by Lorenzo Zambotti and Nikolas Tapia gives a complete parametrisation of Branched Rough Paths. Their renormalisation acts on Branched Rough Paths through the Hairer-Kelly map. We will explain how it can be related to the canonical renormalisation of Branched Rough Paths provided by Hopf Algebras.

7.0.2 Small Ball Probabilities and Metric Entropy for Gaussian Rough Paths, *William Salkeld, Edinburgh*

In this talk, I compute the Small Ball Probabilities of Gaussian Rough Paths. These are intimately connected with the Metric Entropy, a measure of compactness, of the lifted Reproducing Kernel Hilbert space. While many works on Rough Paths study the Large Deviations Principles for Stochastic Processes driven by Gaussian Rough paths, it is a noticeable gap in the literature that Small Ball Probabilities have not been extended to the Rough Path framework.
A Large Deviations Principle provides Macroscopic information about the shape of a distribution. Small Ball Probabilities provide Microscopic information, allowing us to study the optimal approximation of Gaussian Rough paths.
I will also motivate these results with some surprising and useful applications.

7.0.3 The rough signature kernel, *Cristopher Salvi, Oxford*

In the recent paper "Kernels for sequentially ordered data" the authors present a novel framework for kernel learning with sequential data based on the signature kernel. This methodology is put into action in the work " Variational Gaussian Processes with Signature Covariances" where a new variational approach to learn from stream-valued data based on Gaussian processes with signature kernel as covariance function is presented. In the aforementioned results the signature kernel is computed for paths of finite length by truncating the two signatures involved in the inner-product and resorting to the "kernel trick" combined with an extension of the Horner's scheme for polynomial evaluation. In this talk we will show that the un-truncated signature kernel is a solution of a linear hyperbolic PDE which can be efficiently computed by finite differences. Furthermore, we will extend the definition of the signature kernel to the space of geometric rough paths. By a density argument we prove that the newly defined rough signature kernel is a solution of a well-defined rough integral equation and we provide uniform error bounds for its discrete approximation.

8. Further Topics in Stochastic Analysis

8.0.1 Solutions to the Skorokhod embedding problem for n-dimensional Brownian motion and rough paths, *Christina Zou, Oxford*

First formulated in 1954, the Skorokhod embedding problem has become a classical problem in stochastic analysis is: Given a Brownian motion and a probability measure, the task is to stop the trajectories of the process such that the terminal points are distributed according to the given measure. Despite the vast amount of literature on different solutions, most of the results are on the existence and optimality for one-dimensional processes. In this talk, we will present a method to construct the solution for n-dimensional Brownian motion and the enhanced Brownian motion. This talk is based on joint works with Paul Gassiat and Harald Oberhauser.

8.0.2 The multiplicative chaos of fractional Gaussian fields with zero Hurst parameter, *Paul Hager, TU Berlin*

Fractional Brownian motion with a proper normalization converges towards a log-correlated Gaussian process, when the Hurst parameter H tends to zero (see Neuman and Rosenbaum (2018)). We generalize the notion of this normalization and show that it applies to larger class of fractional Gaussian fields (fGf). We later consider the Gaussian multiplicative chaos associated with these fGf. This measure is defined on a domain in $\mathbb{R}^d$ by

$$M_\gamma^H(dx) := e^{\gamma X^H(x) - \frac{\gamma^2}{2} E(X^H(x))^2} dx,$$

where $\{X^H\}_{H\in(0,1)}$ is a sequence of normalized fGfs approaching a log-correlated Gaussian field as $H \to 0$. The $\mathbb{L}^2$-convergence of M_γ^H as $H \to 0$ in the regime $\gamma \leq \sqrt{d}$ follows by standard methods. We show that the elementary approach of Beretycki (2017) can be modified to show this convergence for $\gamma > \sqrt{d}$ when the measures M_γ^H are restricted to the so called "good points" of X^H.

This is a joint work with Eyal Neuman.

8.0.3 Hitting probabilities of a Brownian flow with Radial Drift, *Eyal Neuman, Imperial*

We consider a stochastic flow $\phi_t(x,\omega)$ in $\mathbb{R}^n$ with initial point $\phi_0(x,\omega) = x$, driven by a single n-dimensional Brownian motion, and with an outward radial drift of magnitude $\frac{F(\|\phi_t(x)\|)}{\|\phi_t(x)\|}$, with F nonnegative, bounded and Lipschitz. We consider initial points x lying in a set of positive distance from the origin. We show that there exist constants $C^*, c^* > 0$ not depending on n, such that if $F > C^* n$ then the image of the initial set under the flow has probability 0 of hitting the origin. If $0 \leq F \leq c^* n^{3/4}$, and if the initial set has nonempty interior, then the image of the set has positive probability of hitting the origin.

This is a joint work with Jong Jun Lee and Carl Mueller.

8.0.4 A Case Study on Pareto Optimality for Collaborative Stochastic Games, *Renyuan Xu, Oxford*

Pareto Optimality (PO) is an important concept in game theory to measure global efficiency when players collaborate. In this talk, we start with the PO for a class of continuous-time stochastic games when the number of players is finite. The derivation of PO strategies is based on the formulation and analysis of an auxiliary N-dimensional central controller's stochastic control problem, including its regularity property of the value function and the existence of the solution to the associated Skorokhod problem. This PO strategy is then compared with the set of (non-unique) NEs strategies under the notion of Price of Anarchy (PoA). The upper bond of PoA is derived explicitly in terms of model parameters. Finally, we characterize analytically the precise difference between the PO and the associated McKean-Vlasov control problem with an infinite number of players, in terms of the covariance structure between the optimally controlled dynamics of players and characteristics of the no-action region for the game. This is based on joint work with Xin Guo (UC Berkeley).

9. Participants

Terry Lyons
Peter Friz
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Oleg Butkovsky
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William Salkeld
James Foster
Alexander Shaposhnikov
Francesco Cosentino
Shujian Liao
Mate Gerencser
Renyuan Xu
Adeline Fermanian
Paolo Grazieschi
Remy Messadene
So Takao
Willem van Zuijlen
Trishen Gunaratnam
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Immanuel Zachhuber
Yvain Bruned
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Matteo Burzoni
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Blanka Horvath
Benjamin Seeger
Cristopher Salvi
Lucas BROUX
Emilien BODIOT
Paul Hager
andris Gerasimovics
Jacek Kiedrowski